① الف) $\cos 2x = 3\sin x - 1 \xrightarrow{\text{فرمول طلایی}} 1 - 2\sin^2 x = 3\sin x - 1$ ۲

$\Rightarrow 2\sin^2 x + 3\sin x - 2 = 0 \xRightarrow[\text{روسی}]{\text{روش}} \sin x = \frac{1}{2}$ ✓ , $\sin x = -2$ × غ ق ق

$$\boxed{\Rightarrow x = 2k\pi + \frac{\pi}{6} \quad , \quad x = 2k\pi + \frac{5\pi}{6}}$$

ب) $\cos 2x + \cos x - 2 = 0 \xrightarrow{\text{فرمول طلایی}} 2\cos^2 x + \cos x - 3 = 0 \begin{cases} \rightarrow \boxed{\cos x = 1 \Rightarrow x = 2k\pi} \\ \rightarrow \cos x = -\frac{3}{2} \times \text{غ ق ق} \end{cases}$

② الف) $(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = \sin 4x \Rightarrow -\cos 2x = \sin 4x$

$$\Rightarrow \sin\left(\frac{3\pi}{2} + 2x\right) = \sin 4x \Rightarrow \begin{cases} ①\ 4x = 2k\pi + \frac{3\pi}{2} + 2x \\ ②\ 4x = 2k\pi + \pi - \frac{3\pi}{2} - 2x \end{cases}$$ ۲

$$\boxed{\Rightarrow x = k\pi + \frac{3\pi}{4} \quad , \quad x = \frac{k\pi}{3} - \frac{\pi}{2}}$$

ب) $2\cos^2 x + 2\sin x\cos x = 1 \Rightarrow 1 + \cos 2x + \sin 2x = 1$

$$\Rightarrow \cos 2x = -\sin 2x \Rightarrow \cos 2x = \cos\left(\frac{\pi}{2} + 2x\right) \begin{cases} \rightarrow 2x = 2k\pi + \frac{\pi}{2} + 2x \times \text{غ ق ق} \\ \rightarrow 2x = 2k\pi - \frac{\pi}{2} - 2x \end{cases}$$

$$\boxed{x = \frac{k\pi}{2} - \frac{\pi}{8}} \Leftarrow$$

③ الف) $\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \xrightarrow{\sin x \neq 0} 2\cos^2 x + \cos x - 1 = 0$

$$\Rightarrow \begin{cases} \cos x = -1 \times \rightarrow \text{به دلیل شرط دامنه قابل قبول نیست} \\ \cos x = \frac{1}{2} \quad \Rightarrow \boxed{x = 2k\pi \pm \frac{\pi}{3}} \end{cases}$$ ۲

ب) $2\sin^2 x - 2 + \cos^2 x - \sin x\cos x = 0 \Rightarrow -2\cos^2 x + \cos^2 x - \sin x\cos x = 0$

$$\Rightarrow -\cos x(\cos x + \sin x) = 0 \begin{cases} \cos x = 0 \Rightarrow \boxed{x = k\pi + \frac{\pi}{2}} \\ \cos x = -\sin x \rightarrow \boxed{x = k\pi - \frac{\pi}{4}} \end{cases}$$

Subject ____ Date ____

روشنا ابوسی

(۴

الف) $\cos(\frac{\pi}{4}+x)\times\cos(\frac{\pi}{4}-x)=\frac{1}{4} \Rightarrow \cos(\frac{\pi}{4}+x)\times\sin(\frac{\pi}{4}+x)=\frac{1}{4}$

$\Rightarrow \frac{1}{2}\sin(\frac{\pi}{2}+2x)=\frac{1}{4} \Rightarrow \cos 2x=\frac{1}{2} \Rightarrow 2x=2k\pi\pm\frac{\pi}{3}$ ۱

$\Rightarrow \boxed{x=k\pi\pm\frac{\pi}{6}}$

سوال ۴ قسمت ب

ب)

$\cos(2x-\frac{\pi}{9})=-\sin 2x$

$\cos(2x-\frac{\pi}{9})=\cos(\frac{\pi}{2}+2x)$

$2x-\frac{\pi}{9}=2k\pi\pm(\frac{\pi}{2}+2x)$

حالت + : $2k\pi+\frac{\pi}{9}+\frac{\pi}{2}=0$ ×

حالت − : $2x-\frac{\pi}{9}=2k\pi-\frac{\pi}{2}-2x$

$4x=2k\pi-\frac{7\pi}{18} \rightarrow x=\frac{k\pi}{2}-\frac{7\pi}{72}$

(۵

$\frac{\sin 4x+\sin 2x}{\sin 2x}=1 \Rightarrow \sin 2x\neq 0 \;,\; \sin 4x+\sin 2x=\sin 2x$

$\Rightarrow \sin 4x=0 \;,\; \sin 4x=2\sin 2x\cos 2x$ ۲

$\xrightarrow{\sin 2x\neq 0} \cos 2x=0 \Rightarrow 2x=k\pi+\frac{\pi}{2} \Rightarrow \boxed{x=\frac{k\pi}{2}+\frac{\pi}{4}}$

(۶

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{\cos\frac{x}{2}+1}{\sin\frac{x}{2}} \xrightarrow{\tan x=\frac{\sin 2x}{1+\cos 2x}} \tan\frac{x}{4}=\cot\frac{x}{4}$

$\Rightarrow \frac{x}{4}=\frac{k\pi}{2}+\frac{\pi}{4} \Rightarrow x=2k\pi+\pi$ ۲

$x\in[-\pi,\frac{3\pi}{2}] \rightarrow x=-\pi,\pi \Rightarrow \boxed{|\pi-(-\pi)|=2\pi}$

(۷

$\cos^2 x-\sin^2 x\cos 3x=1 \longrightarrow \cos^2 x-1-\sin^2 x\cos 3x=0$

$\Rightarrow \sin^2 x+\sin^2 x\cos 3x=0 \Rightarrow \sin^2 x(1+\cos 3x)=0$ ۲

$\Rightarrow \begin{cases} \sin^2 x=0 \Rightarrow \sin x=0 \\ 1+\cos 3x=0 \Rightarrow \cos 3x=-1 \Rightarrow 3x=2k\pi+\pi \Rightarrow x=\frac{2k\pi}{3}+\frac{\pi}{3} \end{cases}$

$x\in\{0,\frac{\pi}{3},\pi,\frac{5\pi}{3},2\pi\} \Rightarrow$ پنج جواب دارد

رونیا الیاسی

Subject ______ Date ______

(۸)

$$\tan\left(\frac{\pi}{4}-x\right)=\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}=\frac{1-\tan x}{1+\tan x}$$

۲

$$\Rightarrow \tan\left(\frac{\pi}{4}-x\right)=\tan 3x \Rightarrow 3x=k\pi+\frac{\pi}{4}-x$$

$$\Rightarrow \boxed{x=\frac{k\pi}{4}+\frac{\pi}{16}}$$

(۹)

$$\sqrt{1+2\sin x\cos x}=\sqrt{(\sin x+\cos x)^2}=|\sin x+\cos x|=\sqrt{2}\cos 2x$$

$$\Rightarrow |\sin x+\cos x|=\sqrt{2}(\cos x-\sin x)(\cos x+\sin x)$$

۲

$$\xrightarrow{\sin x+\cos x\neq 0} \sqrt{2}\,|\sin x-\cos x|=1 \Rightarrow \sqrt{2}\,\left|\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)\right|=1$$

$$\Rightarrow 2\left|\sin\left(x-\frac{\pi}{4}\right)\right|=1 \Rightarrow \left|\sin\left(x-\frac{\pi}{4}\right)\right|=\frac{1}{2} \Rightarrow \sin\left(x-\frac{\pi}{4}\right)=\pm\frac{1}{2}$$

$$x-\frac{\pi}{4}=k\pi\pm\frac{\pi}{6} \rightarrow x=k\pi\pm\frac{\pi}{6}+\frac{\pi}{4} \xrightarrow{x\in[0,\pi]}$$ ۲ جواب دارد

(۱۰)

الف) $(\sin^2 x+\cos^2 x)(\sin^2 x-\cos^2 x)=1 \Rightarrow \sin^2 x-\cos^2 x=\sin^2 x+\cos^2 x$

۲

$$\Rightarrow 2\cos^2 x=0 \Rightarrow \cos x=0 \Rightarrow \boxed{x=\frac{\pi}{2},\ x=\frac{3\pi}{2}}$$

ب) $\sin^3 x-\cos^3 x=-1 \underset{\text{اصلی}}{\overset{\text{نقاط}}{\Longrightarrow}} x=0\checkmark,\ x=\frac{\pi}{2}\times,\ x=\pi\times,$

$$x=\frac{3\pi}{2}\checkmark,\ x=2\pi\checkmark$$

$$\Rightarrow \boxed{x\in\left\{0,\frac{3\pi}{2},2\pi\right\}}$$