فرمول کلی $\sin x = \sin\alpha \to x = 2k\pi+\alpha$ ، $x = 2k\pi+\pi-\alpha$

$\cos x = \cos\alpha \to x = 2k\pi\pm\alpha$

۱۸ مرتب تر بنویس!

نام و نام خانوادگی ......صبا امینی...... پاسخنامه تشریحی تکلیف شماره ...۱۶... کلاس ......دوازدهم [illegible]......

جواب آخرها را در اشکال نهایی حتما باید برحسب x حساب کنی مثلا در سوال ۴ برایت جواب درست را نوشتم!

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الف) $\cos 2x = 3\sin x - 1 \to -2\sin^2 x + 1 = 3\sin x - 1$

$\cos 2x \downarrow$ دوالقطبی $-2\sin^2 x + 1$

$0 = 2\sin^2 x + 3\sin x - 2 \xrightarrow{\Delta = b^2-4ac} \frac{-b\pm\sqrt{\Delta}}{2a}$

$\frac{-3\pm\sqrt{9+16}}{4} \to \frac{-3\pm5}{4}$

$\to \frac{1}{2}$ قق ، -2 غق $-1\leq \sin x\leq 1$

$\sin x = \frac{1}{2} \to$ دو الطرفی $\Rightarrow 2k\pi+\frac{\pi}{6}$ ✓ ، $2k\pi+\frac{5\pi}{6}$ ✓

ب) $\cos 2x + \cos x - 2 = 0 \to 2\cos^2 x + \cos x - 3 = 0$

$\cos 2x \downarrow$ دوالقطبی $2\cos^2 x - 1$

$a+b+c=0$ فرمول عمومی: $1 \to$ قق ، $\frac{-3}{2} \to$ غق

$\cos x = 1$ ✓ دو الطرفی $x = 2k\pi$

②

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الف) $\sin^4 x - \cos^4 x = \sin 4x \longrightarrow -\cancel{\cos 2x} = 2\sin 2x\,\cancel{\cos 2x}$

$\underbrace{(\sin^2 x-\cos^2 x)}_{-\cos 2x}\underbrace{(\sin^2 x+\cos^2 x)}_{1}$ ، $\sin 4x = 2\sin 2x\cos 2x$

$\to$ دو $\cos 2x=0$ حذف شد

$-1 = 2\sin 2x \to \sin 2x = -\frac{1}{2}$

$2k\pi\pm\frac{\pi}{2}$ ✓ ، $2k\pi-\frac{\pi}{6}$ ✓ ، $2k\pi-\frac{5\pi}{6}$ ✓

②

$2\cos^2 x + \underbrace{2\sin x\cos x}_{\sin 2x} = 1 \longrightarrow \cancel{1}+\cos 2x+\sin 2x = \cancel{1}$

$2\cos^2 x \downarrow$ ل دوالقطبی $1+\cos 2x$

$\cos 2x = -\sin 2x = \sin(-2x)$

$\sin(\frac{\pi}{2}-2x) = \sin(-2x)$

$-2x = 2k\pi+\frac{\pi}{2}-2x$ ×

$0 = 2k\pi+\frac{\pi}{2}$

$-2x = 2k\pi+\pi-\frac{\pi}{2}+2x$

$x = \frac{2k\pi}{-4}+\frac{\pi}{2\times-4} \to -\frac{k\pi}{2}-\frac{\pi}{8}$ ✓

۳

$\cot x(2\cos x+1) = \frac{1}{\sin x} \xrightarrow{\sin x\neq0} 2\cos^2 x+\cos x-1 = 0$

$\cot x = \frac{\cos x}{\sin x}$

$a+c=b$ عیق $\to \frac{1}{2} \to 2k\pi\pm\frac{\pi}{3}$ ✓

$\to -1 \to 2k\pi+\pi$

اگر $\cos x = -1$ باشد $\sin x = 0$ است و عبارت تعریف نشده می‌شود پس جواب غیر قابل قبول محسوب می‌شود

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$2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \to 1+\sin^2 x - \sin x\cos x = 2$

$\sin^2 x+\sin^2 x$ ، $(=1)$

$\sin^2 x - \sin x\cos x = 1 = \sin^2 x+\cos^2 x$

$\cos(\frac{\pi}{2}-x) \leftarrow \sin x = -\cos x \to = \cos(\pi-x)$

$\to \cos(\frac{\pi}{2}-x) = \cos(\pi-x)$

$\pi-x = 2k\pi\pm(\frac{\pi}{2}-x)$

$-x = 2k\pi+\frac{\pi}{2}-x$ ×

$-2x = 2k\pi-\frac{\pi}{2} \to x = -k\pi+\frac{\pi}{4}$ ✓

۴

$\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4}) = \frac{1}{4} \xrightarrow{\text{زوایا متمم اند}} \frac{\pi}{4}+x+(-x+\frac{\pi}{4}) = \frac{\pi}{2}$ پس

$\cos(x-\frac{\pi}{4}) = \cos(-x+\frac{\pi}{4})$ ، $\cos(\frac{\pi}{4}+x) = \sin(x-\frac{\pi}{4})$

$2x = 2k\pi\pm\frac{\pi}{3} \to x = k\pi\pm\frac{\pi}{6}$

$\frac{1}{2}\sin(2x+\frac{\pi}{2}) = \frac{1}{4} \longrightarrow +\cos 2x = \frac{1}{2}$

$\sin(2x+\frac{\pi}{2}) = -\cos 2x$

$\cos 2x = -\frac{1}{2}$ ✓ $2k\pi\pm\frac{2\pi}{3}$

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$\cos(2x-\frac{\pi}{9}) = -\sin 3x \to -(\cos(\frac{\pi}{2}-3x)) = \cos(\frac{\pi}{2}+3x)$

$2x-\frac{\pi}{9} = 2k\pi\pm(\frac{\pi}{2}-3x)$

$\to 2x-\frac{\pi}{9} = 2k\pi+\frac{\pi}{2}-3x \to 5x = 2k\pi+\frac{11\pi}{18}$

$\to 2x-\frac{\pi}{9} = 2k\pi-\frac{\pi}{2}+3x \to 0 = 2k\pi-\frac{7\pi}{18}$ × ✓

$x = \frac{2k\pi}{5}-\frac{11\pi}{90}$

۵

$\frac{\sin 4x+\sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \xrightarrow{\sin^2 x+\cos^2 x=1} \frac{2\sin 2x\cos 2x}{\sin 2x} = 1$

$\sin 2x\neq0$

$\sin 4x = 2\sin 2x\cos 2x$

$\cos 2x = \frac{1}{2} \leftarrow 1 = 2\cos 2x$

$\cos 2x = \frac{1}{2} \longrightarrow \cos 2x = \cos(\frac{\pi}{3}) \to 2x = 2k\pi\pm\frac{\pi}{3}$

$x = k\pi\pm\frac{\pi}{6}$

$\sin 2x\neq0$

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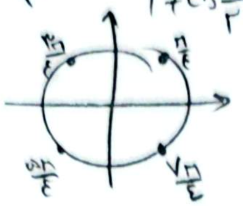
۶

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} \qquad \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \to \tan\frac{x}{4} = \frac{1}{\tan\frac{x}{4}} \to \tan^2\frac{x}{4} = 1$$

$\tan\frac{x}{4} = \pm 1$



$\frac{x}{4} \in \frac{k\pi}{2} \pm \frac{\pi}{4} \to x \in 2k\pi \pm \pi$

$-\pi < x \leq \frac{3\pi}{2}$ $\to x = -\pi, \pi$

تفاضل $= 2\pi$ ✓

۲

۷

$\cos^2 x - \sin^2 x \cos^3 x = 1 = \sin^2 x + \cos^2 x \to -\sin^2 x \cos^3 x = \sin^2 x$ حذف شدن $\to \sin^2 x = 0$

$\sin x = 0$ ✓

[illegible]

$-\cos^3 x = 1 \to \cos^3 x = -1$

$3x = 2k\pi + \pi$

$x = \frac{2k\pi}{3} + \frac{\pi}{3}$

$k = 0 \to \frac{\pi}{3}$

$k = 1 \to \pi$

$k = 2 \to \frac{5\pi}{3}$

} جواب غیر تکراری

تکراری

۵ جواب

۴ جواب

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۸

$$\tan 3x = \frac{1-\tan x}{1+\tan x} \xrightarrow{\times\frac{1-\tan}{1-\tan x}} \frac{(1-\tan x)^2}{1-\tan^2 x} = \tan 3x$$

$$\frac{\tan\frac{\pi}{4} - \tan x}{1+(\tan\frac{\pi}{4} \times \tan x)} = \tan 3x \to \tan\left(\frac{\pi}{4} - x\right) = \tan 3x \to 3x = k\pi + \frac{\pi}{4} - x$$

$x = \frac{k\pi}{4} + \frac{\pi}{16}$ ✓ $\leftarrow 4x = k\pi + \frac{\pi}{4}$

۲

۹

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\cos 2x > 0} 1 + \sin 2x = 2\underbrace{\cos^2 2x}_{(1-\sin^2 2x)}$$

$\cancel{1} + \sin 2x = \cancel{2} - \sin^2 2x$

$\sin^2 2x + \sin 2x = 0$

$\sin 2x(\sin 2x + 1) = 0$

$\to \sin 2x = 0 \quad 2x = k\pi \to x = \frac{k\pi}{2} \to \frac{\pi}{2}, \pi$

۳ جواب

$\to \sin 2x + 1 = 0$

$\sin 2x = -1 \to 2x = 2k\pi + \frac{3\pi}{2} \to x = k\pi + \frac{3\pi}{4} \to \frac{3\pi}{4}$

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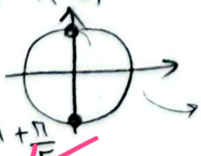
۱۰

$$\sin^4 x - \cos^4 x = 1 \to (\sin^2 x - \underbrace{\cos^2 x}_{(1-\sin^2 x)})(\underbrace{\sin^2 x + \cos^2 x}_{1}) = 1$$

$\sin^2 x - 1 + \sin^2 x = 1 \to 2\sin^2 x - 2 = 0$ ✓ $\quad \sin^2 x = 1$

$\sin x = \pm 1$



$\to \frac{\pi}{2}, \frac{3\pi}{2}$

$\hookrightarrow x = k\pi + \frac{\pi}{2}$ ✓

۲

$\sin^3 x - \cos^3 x = -1$

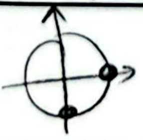
$(\sin x - \cos x)(\underbrace{\sin^2 x + \cos^2 x}_{1} + \underbrace{\sin x \cos x}_{\frac{1}{2}\sin 2x}) = -1$ ↗ از ۴ نقطه اصلی

۰ درجه $\to (0-1)(1+0) = -1$ قق ✓

۹۰ درجه $\to (1)(1+0) = 1$ غق ✗

۱۸۰ درجه $\to (0+1)(1+0) = 1$ غق ✗

۲۷۰ درجه $\to (-1+0)(1+0) = -1$ قق ✓



$2\pi / \frac{3\pi}{2} / 0 \leftarrow$ جواب‌ها ✓

سوال ۹

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$$

$$1+\sin 2x = 2-2\sin^2 2x \to 2\sin^2 2x + \sin 2x - 1 = 0 \quad \begin{cases} \sin 2x = -1 \\ \sin 2x = +\frac{1}{2} \end{cases}$$

$$\sin 2x = -1 \to 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0 \le x \le \pi} x = \frac{3\pi}{4} \; ✓$$

$$\sin 2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0 \le x \le \pi} x = \frac{\pi}{12} \; ✓ \text{ یا } \frac{5\pi}{12} \; ✗$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x = \frac{5\pi}{12}$ ، $\cos 2x$ منفی می‌شود، چون جواب رادیکال است غ‌ق‌ق است

* معادله ۲ جواب دارد

سوال ۷

$$\cos^2 x - \sin^2 x \cos 3x = 1 \xrightarrow{\cos^2 x = 1 - \sin^2 x}$$

$$\cancel{1} - \sin^2 x - \sin^2 x \cos 3x = \cancel{1} \to \sin^2 x(1+\cos 3x) = 0 \quad \begin{cases} \sin x = 0 \\ \cos 3x = -1 \end{cases}$$

$$\sin x = 0 \to x = k\pi \to x = \overset{③}{0}, \overset{②}{\pi}, \overset{①}{2\pi}$$

$$\cos 3x = -1 \to 3x = 2k\pi + \pi \to x = \frac{2k\pi + \pi}{3} \to x = \overset{⑤}{\frac{\pi}{3}}, \pi, \overset{④}{\frac{5\pi}{3}}$$

معادله ۵ جواب دارد

سوال ۵

$$\frac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x} = \overset{1}{\cancel{\sin^2 x + \cos^2 x}}$$

$$\frac{\cancel{\sin 2x}(2\cos 2x + 1)}{\cancel{\sin 2x}} = 1 \xrightarrow{\sin 2x \neq 0} 2\cos 2x + 1 = 1 \to \cos 2x = 0$$

$$2x = k\pi + \frac{\pi}{2} \to \boxed{x = k\pi + \frac{\pi}{4}}$$

سوال۴ قسمت الف)

$\cos(-x) = \cos x$

$\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4}) = \frac{1}{4} = \cos(\frac{\pi}{4}+x)\cos(\frac{\pi}{4}-x) = \frac{1}{4}$

$\text{if } \alpha+\beta = \frac{\pi}{2} \rightarrow \cos\alpha = \sin\beta \rightarrow \cos(\frac{\pi}{4}+x)\sin(\frac{\pi}{4}+x) = \frac{1}{4}$

$\frac{1}{2}\sin(\frac{\pi}{2}+2x) = \frac{1}{4} \rightarrow \cos 2x = \frac{1}{2} \rightarrow 2x = 2k\pi \pm \frac{\pi}{3}$

$$\boxed{x = k\pi \pm \frac{\pi}{6}}$$