نکته: $\sin x = \sin\alpha \to x = 2k\pi + \alpha$ ، $x = 2k\pi + \pi - \alpha$ ؛ $\cos x = \cos\alpha \to x = 2k\pi \pm \alpha$

نام و نام خانوادگی .... **پاسخنامه تشریحی تکلیف شماره** ...۱۶... **کلاس** ....

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الف) $\cos 2x = 3\sin x - 1 \to -2\sin^2 x + 1 = 3\sin x - 1$

طبق فرمول: $\cos 2x = -2\sin^2 x + 1$

$0 = 2\sin^2 x + 3\sin x - 2 \xrightarrow{\Delta = b^2 - 4ac} \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4}$

-2 غ ق ق ($-1 \le \sin x \le 1$) ؛ $\frac{1}{2}$

$\sin x = \frac{1}{2} \to x = 2k\pi + \frac{\pi}{6}$ ، $2k\pi + \frac{5\pi}{6}$

ب) $\cos 2x + \cos x - 2 = 0 \to 2\cos^2 x + \cos x - 3 = 0$

طبق فرمول: $\cos 2x = 2\cos^2 x - 1$

1 ق ق ؛ $-\frac{3}{2}$ غ ق ق

$\cos x = 1$ در نتیجه $x = 2k\pi$

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الف) $\sin^4 x - \cos^4 x = \sin 4x$

$(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = -\cos 2x \cdot 1$ ، $\sin 4x = 2\sin 2x\cos 2x$

$\to -\cos 2x = 2\sin 2x\cos 2x$ ($\cos 2x = 0$ قابل قبول)

$-1 = 2\sin 2x \to \sin 2x = -\frac{1}{2}$ → $2x = 2k\pi - \frac{\pi}{6}$ ، $2k\pi - \frac{5\pi}{6}$

$2\cos^2 x + 2\sin x\cos x = 1 \to 1 + \cos 2x + \sin 2x = 1$

$\cos 2x = -\sin 2x = \sin(-2x)$

$\sin(\frac{\pi}{2} - 2x) = \sin(-2x)$

$-2x = 2k\pi + \frac{\pi}{2}$ ... $\times 0 = 2k\pi + \frac{\pi}{2}$ ✗

$-2x = 2k\pi + \pi - \frac{\pi}{2} + 2x$

$x = \frac{2k\pi}{-4} + \frac{\pi}{2\times(-4)} \to x = -\frac{k\pi}{2} - \frac{\pi}{8}$

۳

$\cot x(2\cos x + 1) = \frac{1}{\sin x}$ ، $\cot x = \frac{\cos x}{\sin x}$ ، $\sin x \ne 0$

$\to 2\cos^2 x + \cos x - 1 = 0$

$a + c = b$: $\frac{1}{2} \to 2k\pi \pm \frac{\pi}{3}$ ؛ $-1 \to 2k\pi + \pi$

$2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \to 1 + \sin^2 x - \sin x\cos x = 2$

$\sin^2 x - \sin x\cos x = 1 = \sin^2 x + \cos^2 x$

$\sin x = -\cos x = \cos(-x)$ → $\cos(\frac{\pi}{2} - x)$

$\cos(\frac{\pi}{2} - x) = \cos(-x)$

$-x = 2k\pi \pm (\frac{\pi}{2} - x)$

$-x = 2k\pi + \frac{\pi}{2} - x$ ✗

$-2x = 2k\pi - \frac{\pi}{2} \to x = -k\pi + \frac{\pi}{4}$

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$\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \frac{1}{4}$

$\cos(x - \frac{\pi}{4}) = \cos(-x + \frac{\pi}{4})$

زوایا متمم اند: $\frac{\pi}{4} + x + (-x + \frac{\pi}{4}) = \frac{\pi}{2}$ پس $\cos(\frac{\pi}{4} + x) = \sin(x - \frac{\pi}{4})$

$\frac{1}{2}\sin(2x - \frac{\pi}{2}) = \frac{1}{4} \to -\cos 2x = \frac{1}{2}$

$\sin(2x - \frac{\pi}{2}) = -\cos 2x$

$\cos 2x = -\frac{1}{2} \to 2x = 2k\pi \pm \frac{2\pi}{3}$

$\cos(2x - \frac{\pi}{9}) = -\sin 2x$ → $-(\cos(\frac{\pi}{2} - 2x)) = \cos(\frac{\pi}{2} - 2x)$

$2x - \frac{\pi}{9} = 2k\pi \pm (\frac{\pi}{2} - 2x)$

$\to 2x - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} - 2x \to x = \frac{k\pi}{2} + \frac{11\pi}{72}$

$\to 2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} + 2x \to 0 = 2k\pi - \frac{7\pi}{18}$ ✗

۵

$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \xrightarrow{\sin^2 x + \cos^2 x = 1} \frac{2\sin 2x\cos 2x}{\sin 2x} = 1$

$\sin 2x \ne 0$ ، $\sin 4x = 2\sin 2x\cos 2x$

$1 = 2\cos 2x \to \cos 2x = \frac{1}{2}$

$\cos 2x = \frac{1}{2} \to \cos 2x = \cos(\frac{\pi}{3}) \to 2x = 2k\pi \pm \frac{\pi}{3}$

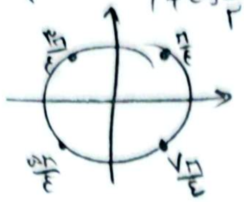
$x = k\pi \pm \frac{\pi}{6}$

$\sin 2x \ne 0$

۶

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} \qquad \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \to \tan\frac{x}{4} = \frac{1}{\tan\frac{x}{4}} \to \tan^2\frac{x}{4} = 1$$

$\tan\frac{x}{4} = \pm 1$



$\frac{x}{4} \in \frac{k\pi}{2} \pm \frac{\pi}{4} \to x \in 2k\pi \pm \pi$

$-\pi < x \le \frac{3\pi}{2}$ → $x = -\pi, \pi$

3π = نقض

۷

$\cos^2 x - \sin^2 x \cos 3x = 1 = \sin^2 x + \cos^2 x \to -\sin^2 x \cos 3x = \sin^2 x$ حذف نکن → $\sin^2 x = 0$, $\sin x = 0$ $\{0, \pi, 2\pi\}$

$-\cos 3x = 1 \to \cos 3x = -1$

$3x = 2k\pi + \pi$

$x = \frac{2k\pi}{3} + \frac{\pi}{3}$

$k=0 \to \frac{\pi}{3}$

$k=1 \to \pi$, $k=2 \to \frac{5\pi}{3}$ } ۳ جواب غیرتکراری → ۵ جواب

تکراری

۸

$$\tan 3x = \frac{1-\tan x}{1+\tan x} \xrightarrow{\times \frac{1-\tan}{1-\tan x}} \frac{(1-\tan x)^2}{1-\tan^2 x} = \tan 3x$$

$$\frac{\tan\frac{\pi}{4} - \tan x}{1+(\tan\frac{\pi}{4} \times \tan x)} = \tan 3x \to \tan\left(\frac{\pi}{4} - x\right) = \tan 3x \to 3x = k\pi + \frac{\pi}{4} - x$$

$x = \frac{k\pi}{4} - \frac{\pi}{16} \leftarrow 4x = k\pi + \frac{\pi}{4}$

۹

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\cos 2x > 0} 1 + \sin 2x = 2\cos^2 2x$ $(1-\sin^2 2x)$

$1 + \sin 2x = 2 - \sin^2 2x$

$\sin^2 2x + \sin 2x = 0$

$\sin 2x(\sin 2x + 1) = 0$ → $\sin 2x = 0$, $2x = k\pi \to x = \frac{k\pi}{2} \to \frac{\pi}{2}, \pi$ ۳ جواب

→ $\sin 2x + 1 = 0$

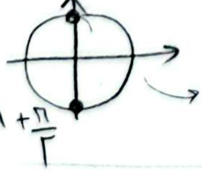
$\sin 2x = -1 \to 2x = 2k\pi + \frac{3\pi}{2} \to x = k\pi + \frac{3\pi}{4}$ → $\frac{3\pi}{4}$

۱۰

$\sin^4 x - \cos^4 x = 1 \to (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = 1$

$(1-\sin^2 x)$; $\sin^2 x - 1 + \sin^2 x = 1 \to 2\sin^2 x - 2 = 0$ $\sin^2 x = 1$

$\sin x = \pm 1$



→ $\frac{\pi}{2}, \frac{3\pi}{2}$

$x = k\pi + \frac{\pi}{2}$

$\sin^3 x - \cos^3 x = -1$

$(\sin x - \cos x)(\sin^2 x + \cos^2 x + \sin x\cos x) = -1$ ← از نقاط اصلی

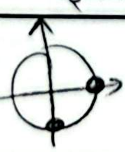
1 ; $\frac{1}{2}\sin 2x$

✓ قق ۱- ۰درجه → $(0-1)(1+0) = -1$

× قق ۱ ۹۰درجه → $(1)(1+0) = 1$

× قق ۱ ۱۸۰درجه → $(0+1)(1+0) = 1$

✓ قق ۱- ۲۷۰درجه → $(-1+0)(1+0) = -1$



جوابها ← $0 / \frac{3\pi}{2} / 2\pi$