۱)

الف) $\underset{1-2\sin^2 x}{\cos 2x} = 3\sin x - 1 \rightarrow 2\sin^2 x + 3\sin x - \cancel{2}$ -4

$(\sin x + 4)(\sin x - 1) \rightarrow \sin x = \frac{1}{2}$

$-\frac{4}{2}$ -2 غقق $\frac{1}{2}$

۲۰ آفرین خیلی عالی نوشتی :)

$\begin{cases} x = 2k\pi + \frac{\pi}{6} \\ x = 2k\pi + \frac{5\pi}{6} \end{cases}$ $k \in \mathbb{Z}$

۲

ب) $\underset{2\cos^2 x - 1}{\cos 2x} + \cos x - 2 = 0 \rightarrow 2\cos^2 x + \cos x - 3 = 0$

جمع ضرایب $= 0 \rightarrow$ ریشه‌ها 1 و $\frac{c}{a} = \cancel{\frac{-3}{2}}$ غقق

$\cos x = 1$

$x = 2k\pi$ $k \in \mathbb{Z}$

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۲)

الف) $\underset{\sin^2 x - \cos^2 x}{\cancel{\sin^4 x - \cos^4 x}} = \sin 4x \rightarrow -\cancel{\cos 2x} = 2\cancel{\cos 2x} \sin 2x$

$\sin 2x = -\frac{1}{2}$ $\cos 2x = 0$ *

$\hookrightarrow 2x = 2k\pi - \frac{\pi}{6} \Rightarrow x = k\pi - \frac{\pi}{12}$

$2x = 2k\pi - \frac{5\pi}{6} \Rightarrow x = k\pi - \frac{5\pi}{12}$

۲

* $\cos 2x = 0 \rightarrow 2x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$ $k \in \mathbb{Z}$

ب) $2\cos^2 x + 2\sin x\cos x = 1 \rightarrow \underset{\sin 2x}{\cancel{2\sin x\cos x}} = \underset{\underset{\sin(\frac{\pi}{2} - 2x)}{-\cos 2x}}{\cancel{1 - 2\cos^2 x}}$

$\rightarrow \sin 2x = \sin(2x - \frac{\pi}{2})$

$\cancel{2x - \frac{\pi}{2} = 2k\pi + 2x}$

$2x - \frac{\pi}{2} = 2k\pi + \pi - 2x \rightarrow 4x = 2k\pi + \frac{3\pi}{2}$

$x = \frac{k\pi}{2} + \frac{3\pi}{8}$ $k \in \mathbb{Z}$

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۳)

الف) $\underset{\cancel{\frac{\cos x}{\sin x}}}{\cot x}(2\cos x + 1) = \cancel{\frac{1}{\sin x}} \rightarrow 2\cos^2 x + \cos x - 1 = 0$

$a + c = b \rightarrow$ ریشه‌ها -1 و $-\frac{c}{a} = \frac{1}{2}$

۲

$\cos x = -1 \rightarrow x = 2k\pi + \pi \longrightarrow$ غقق چون $\sin x \neq 0$

$\cos x = \frac{1}{2} \rightarrow x = 2k\pi \pm \frac{\pi}{3}$ $k \in \mathbb{Z}$

ب) $\underset{2(-\cos^2 x)}{2\sin^2 x} - \sin x\cos x + \cos^2 x = \cancel{2}$ $\rightarrow -\sin x\cos x = \cos^2 x$

$\cos x = 0$

$x = k\pi + \frac{\pi}{2}$ $k \in \mathbb{Z}$

$\underset{\sin(-x)}{-\sin x} = \sin(\frac{\pi}{2} - x)$ $\cancel{-x = 2k\pi + \frac{\pi}{2} - x}$

$-x = 2k\pi + \pi - \frac{\pi}{2} + x \rightarrow -2x = 2k\pi + \frac{\pi}{2}$

$x = -k\pi - \frac{\pi}{4}$ $k \in \mathbb{Z}$

الف) $\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4})=\frac{1}{4}$

$\cos(\frac{\pi}{4}-x) \quad \sin(\frac{\pi}{4}+x)$

دو زاویه متمم یکدیگرند

$\cos(\frac{\pi}{4}+x)\sin(\frac{\pi}{4}+x)=\frac{1}{4}$

$\frac{1}{\cancel{2}}\sin(2x+\frac{\pi}{2})=\frac{1}{\cancel{4}2}$

$\cos 2x$

۲

$\cos 2x=\frac{1}{2}\rightarrow 2x=2k\pi\pm\frac{\pi}{3}$

$\boxed{x=k\pi\pm\frac{\pi}{6}} \mid k\in\mathbb{Z}$

ب) $\cos(2x-\frac{\pi}{9})=-\sin 2x$

$-\cos(\frac{\pi}{2}-2x) \xrightarrow{\times(-1)} \cos(\frac{\pi}{9}-2x)=\cos(\frac{\pi}{2}-2x)$

$\cancel{\frac{\pi}{9}-2x=2k\pi+\frac{\pi}{2}-2x}$

$\frac{\pi}{9}-2x=2k\pi+\underbrace{\pi-\frac{\pi}{2}}_{\frac{\pi}{2}}+2x$

$-4x=2k\pi-\frac{7\pi}{18}$

$\boxed{x=-\frac{k\pi}{2}+\frac{7\pi}{72}} \quad k\in\mathbb{Z}$

۵

$\frac{\sin 4x+\sin 2x}{\sin 2x}$ $\quad (1-\sin^2 x=\cos^2 x)$ ①

$\frac{\sin 4x+\sin 2x}{\sin 2x}=\frac{1}{1}$

۲

$\sin 2x=\sin 4x+\cancel{\sin 2x}\rightarrow \sin 4x=0$

$k\in\mathbb{Z}$

$4x=2k\pi \rightarrow x=\frac{k\pi}{2}$

$4x=2k\pi+\pi \rightarrow x=\frac{k\pi}{2}+\frac{\pi}{4}$

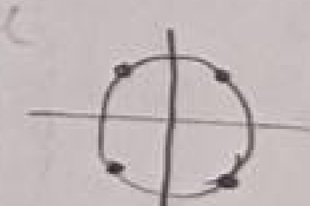
۶

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1}{\sin\frac{x}{2}}+\cot\frac{x}{2} \quad \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}=\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$

۲

$\frac{\cancel{2}\sin\frac{x}{4}\cos\frac{x}{4}}{\cancel{2}\cos^2\frac{x}{4}}=\frac{\cancel{2}\cos^2\frac{x}{4}}{\cancel{2}\sin\frac{x}{4}\cos\frac{x}{4}}$

$\tan\frac{x}{4}=\cot\frac{x}{4}$



$\frac{x}{4}=\frac{k\pi}{2}+\frac{\pi}{4}\rightarrow \boxed{x=2k\pi+\pi}$

$k\in\mathbb{Z}$

در بازه $[-\pi , \frac{3\pi}{2}]$ $\rightarrow x=-\pi , \pi \xrightarrow{\text{قدر مطلق تفاضل}} |-\pi-\pi|=2\pi$

۷

$\cancel{\cos^2 x} - \sin^2 x \cos 3x = \cancel{1}\ 0$

$-\sin^2 x$

$-\underbrace{\sin x}_{0}(1+\underbrace{\cos 3x}_{-1})=0$ **۲**

$x = k\pi$ $k\in\mathbb{Z}$

$\cancel{3}x = \boxed{\frac{2k\pi}{3}+\frac{\pi}{3}}$ $k\in\mathbb{Z}$

تعداد جواب‌ها در $[0, 2\pi]$: $0, \pi, \frac{\pi}{3}, \cancel{\frac{2\pi}{3}+\frac{\pi}{3}}, \frac{5\pi}{3}, 2\pi$ ۵ جواب

۸

$\frac{\tan\frac{\pi}{4} - \tan x}{1+\tan\frac{\pi}{4}\tan x} = \tan 3x$

$\tan(\frac{\pi}{4}-x)$

$\tan(\frac{\pi}{4}-x) = \tan 3x$ **۲**

$3x = k\pi + \frac{\pi}{4} - x$

$4x$ $\to \boxed{x = \frac{k\pi}{4}+\frac{\pi}{16}}$ $x\in\mathbb{Z}$

۹

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x$

$\cos(\frac{\pi}{2}-2x)$

$\sqrt{2\cos^2(\frac{\pi}{4}-x)}$ $\quad \sqrt{2}\cos(\frac{\pi}{4}-x)$

$\cos(\frac{\pi}{4}-x) = \cos 2x$ **۲**

$k\in\mathbb{Z}$

$2x = 2k\pi + \frac{\pi}{4} - x \to x = \frac{2k\pi}{3}+\frac{\pi}{12}$

$2x = 2k\pi - \frac{\pi}{4} + x \to x = 2k\pi - \frac{\pi}{4}$

تعداد جواب در $[0, \pi]$: $\frac{\pi}{12}$ و $\frac{9\pi}{12}$ $\to$ ۲ جواب

۱۰

الف) $\sin^4 x - \cos^4 x = 1$

$\sin^2 x - \cos^2 x$ $\to$ بررسی ۴ نقطه‌ی اصلی

$\to \cancel{\sin^2 x - \cos^2 x = -1}$

$\to \sin^2 x - \cos^2 x = 1$

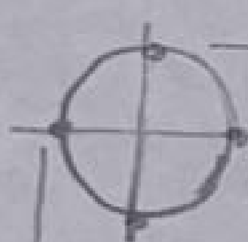
$\to \sin^2 x - \cos^2 x = -1$ ✓

$\to \sin^2 x - \cos^2 x = 1$ ✓

$\boxed{x = \frac{\pi}{2} \text{ و } \frac{3\pi}{2}}$

جواب‌ها در بازه‌ی $[0, 2\pi]$ **۲**

ب) $\sin^3 x - \cos^3 x = -1$ $\to$ بررسی ۴ نقطه‌ی اصلی



$\to \cancel{\sin^3 x - \cos^3 x = 1}$

$\to \sin^3 x - \cos^3 x = -1$ ✓

$\to \sin^3 x - \cos^3 x = -1$ ✓

$\to \cancel{\sin^3 x - \cos^3 x = 1}$

$\boxed{x = 0, 2\pi, \frac{3\pi}{2}}$

جواب‌ها در بازه‌ی $[0, 2\pi]$