1

الف) $\cos 2x = 3\sin x - 1$ ، $1 - 2\sin^2 x$ $\rightarrow 2\sin^2 x + 3\sin x - \cancel{2}$ ، -4

$(\sin x + 4)(\sin x - 1) \rightarrow \sin x = \frac{1}{2}$

$-\cancel{\frac{4}{2}}\ -2$ غق ، $\frac{1}{2}$

$$\begin{cases} x = 2k\pi + \frac{\pi}{6} \\ x = 2k\pi + \frac{5\pi}{6} \end{cases} \quad k \in \mathbb{Z}$$

ب) $\cos 2x + \cos x - 2 = 0$ ، $2\cos^2 x - 1$ $\rightarrow 2\cos^2 x + \cos x - 3 = 0$

جمع ضرایب $= 0 \xrightarrow{\text{ریشه‌ها}} 1$ و $\frac{c}{a} = \cancel{\frac{-3}{2}}$ غق

$\cos x = 1$

$x = 2k\pi \quad k \in \mathbb{Z}$

2

الف) $\cancel{\sin^4 x - \cos^4 x} = \sin 4x$ ، $\sin^2 x - \cos^2 x$ $\rightarrow -\cancel{\cos 2x} = 2\cancel{\cos 2x}\sin 2x$

$\sin 2x = \frac{-1}{2}$ ، $\cos 2x = 0$ *

$\hookrightarrow 2x = 2k\pi - \frac{\pi}{6} \Rightarrow x = k\pi - \frac{\pi}{12}$

$2x = 2k\pi - \frac{5\pi}{6} \Rightarrow x = k\pi - \frac{5\pi}{12}$

* $\cos 2x = 0 \rightarrow 2x = k\pi \pm \frac{\pi}{2} \rightarrow \boxed{x = \frac{k\pi}{2} \pm \frac{\pi}{4}}$ $k \in \mathbb{Z}$

ب) $2\cos^2 x + 2\sin x\cos x = 1 \rightarrow \cancel{2\sin x\cos x} = \cancel{1 - 2\cos^2 x}$

$\sin 2x$ ، $-\cos 2x$ ، $\sin(\frac{\pi}{2} - 2x)$

$\rightarrow \sin 2x = \sin(2x - \frac{\pi}{2})$

$\cancel{2x - \frac{\pi}{2} = 2k\pi + 2x}$

$2x - \frac{\pi}{2} = 2k\pi + \pi - 2x \rightarrow 4x = 2k\pi + \frac{3\pi}{2}$

$\boxed{x = \frac{k\pi}{2} + \frac{3\pi}{8}}$ $k \in \mathbb{Z}$

3

الف) $\cot x\,(2\cos x + 1) = \frac{1}{\cancel{\sin x}}$ ، $\frac{\cos x}{\cancel{\sin x}}$ $\rightarrow 2\cos^2 x + \cos x - 1 = 0$

$a + c = b \xrightarrow{\text{ریشه‌ها}} -1$ و $-\frac{c}{a} = \frac{1}{2}$

$\cos x = -1 \rightarrow x = 2k\pi + \pi \longrightarrow$ غق چون $\sin x \neq 0$

$\cos x = \frac{1}{2} \rightarrow \boxed{x = 2k\pi \pm \frac{\pi}{3}}$ $k \in \mathbb{Z}$

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2$

$2(-\cos^2 x)$ ، $\cancel{-2}$ $\rightarrow -\sin x\cos x = \cancel{\cos^2 x}$

$\cos x = 0$

$x = k\pi + \frac{\pi}{2}$ $k \in \mathbb{Z}$

$-\sin x = \sin(\frac{\pi}{2} - x)$ ، $\sin(-x)$

$\cancel{-x = 2k\pi + \frac{\pi}{2} - x}$

$-x = 2k\pi + \pi - \frac{\pi}{2} + x \rightarrow -2x = 2k\pi + \frac{\pi}{2}$

$x = -k\pi - \frac{\pi}{4}$ $k \in \mathbb{Z}$

۴

الف) $\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4}) = \frac{1}{4}$

$\cos(\frac{\pi}{4}-x) = \sin(\frac{\pi}{4}+x)$

دو زاویه متمم یکدیگرند

$\cos(\frac{\pi}{4}+x)\sin(\frac{\pi}{4}+x) = \frac{1}{4}$

$\frac{1}{2}\sin(2x+\frac{\pi}{2}) = \frac{1}{4}$

$\cos 2x$

$\cos 2x = \frac{1}{2} \rightarrow 2x = 2k\pi \pm \frac{\pi}{3}$

$\boxed{x = k\pi \pm \frac{\pi}{6}} \mid k\in \mathbb{Z}$

ب) $\cos(2x-\frac{\pi}{9}) = -\sin 2x = -\cos(\frac{\pi}{2}-2x)$

$\xrightarrow{\times(-1)} \cos(\frac{\pi}{9}-2x) = \cos(\frac{\pi}{2}-2x)$

$\cancel{\frac{\pi}{9}-2x = 2k\pi + \frac{\pi}{2}-2x}$

$\frac{\pi}{9}-2x = 2k\pi + \underbrace{\pi-\frac{\pi}{2}}_{\frac{\pi}{2}} + 2x$

$-4x = 2k\pi - \frac{7\pi}{18}$

$\boxed{x = -\frac{k\pi}{2} + \frac{7\pi}{72}}$ $k\in \mathbb{Z}$

۵

$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x$ ①

$\frac{\sin 4x + \sin 2x}{\sin 2x} = \frac{1}{1}$

$\sin 2x = \sin 4x + \cancel{\sin 2x} \rightarrow \sin 4x = 0$

$k\in \mathbb{Z}$

$4x = 2k\pi \rightarrow x = \frac{k\pi}{2}$

$4x = 2k\pi + \pi \rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$

۶

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2}$ $\quad \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}$ $= \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$

$\frac{\overset{\cancel{2}\sin\frac{x}{4}\cos\frac{x}{4}}{\sin\frac{x}{2}}}{\cancel{2}\cos^2\frac{x}{4}} = \frac{\cancel{2}\cos^2\frac{x}{4}}{\underset{\cancel{2}\sin\frac{x}{4}\cos\frac{x}{4}}{\sin\frac{x}{2}}}$

$\tan\frac{x}{4} = \cot\frac{x}{4}$

$\frac{x}{4} = \frac{k\pi}{2} + \frac{\pi}{4} \rightarrow \boxed{x = 2k\pi + \pi}$

$k\in \mathbb{Z}$

$\xrightarrow{\text{در بازه}} [-\pi , \frac{3\pi}{2}]$ $\quad x = -\pi , \pi \xrightarrow{\text{قدرمطلق تفاضل}} |-\pi-\pi| = 2\pi$

$\cancel{\cos^2 x} - \sin^2 x \cos 3x = \cancel{1}\ 0$

$-\sin^2 x$

۷

$-\underbrace{\sin x}_{0}(1+\underbrace{\cos 3x}_{-1})=0$

$x = k\pi \quad k\in\mathbb{Z}$

$\cancel{3}x = \boxed{\frac{2k}{3}\pi + \frac{\pi}{3}} \quad k\in\mathbb{Z}$

تعداد جواب ها

در $[0, 2\pi]$ $\overrightarrow{\quad}$ 0 و π و 2π و $\frac{\pi}{3}$ و $\cancel{\frac{4\pi}{3}+\frac{\pi}{3}}$ $\frac{5\pi}{3}$ $\}$ ۵ جواب

۸

$\frac{\tan\frac{\pi}{4} - \tan x}{1+\tan x} = \tan 3x$

$\underbrace{\quad}_{\tan\left(\frac{\pi}{4}-x\right)}$

$\tan\left(\frac{\pi}{4}-x\right) = \tan 3x$

$3x = k\pi + \frac{\pi}{4} - x \rightarrow \boxed{x = \frac{k\pi}{4} + \frac{\pi}{16}} \quad x\in\mathbb{Z}$

$4x$

۹

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x$

$\underbrace{\quad}_{\cos\left(\frac{\pi}{2}-2x\right)}$

$\sqrt{2\cos^2\left(\frac{\pi}{4}-x\right)} \quad \sqrt{2}\cos\left(\frac{\pi}{4}-x\right)$

$\cos\left(\frac{\pi}{4}-x\right) = \cos 2x \quad k\in\mathbb{Z}$

$2x = 2k\pi + \frac{\pi}{4} - x \rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{12}$

$2x = 2k\pi - \frac{\pi}{4} + x \rightarrow x = 2k\pi - \frac{\pi}{4}$

تعداد جواب

در $[0, \pi]$ $\overrightarrow{\quad}$ $\frac{\pi}{12}$ و $\frac{9\pi}{12}$ $\longrightarrow$ $\boxed{\text{۲ جواب}}$

۱۰

الف) $\sin^4 x - \cos^4 x = 1$

$\sin^2 x - \cos^2 x$

بررسی انتهای اصلی

$\rightarrow \cancel{\sin^2 x - \cos^2 x = -1}$

$\rightarrow \sin^2 x - \cos^2 x = 1$

$\rightarrow \cancel{\sin^2 x - \cos^2 x = -1}$ ✓

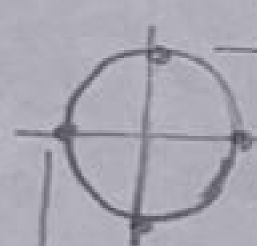
$\sin^2 x - \cos^2 x = 1$ ✓

$\boxed{x = \frac{\pi}{2} \text{ و } \frac{3\pi}{2}}$

جواب ها در بازه ی $[0, 2\pi]$

ب) $\sin^3 x - \cos^3 x = -1$

بررسی انتهای اصلی



$\rightarrow \cancel{\sin^3 x - \cos^3 x = 1}$

$\rightarrow \sin^3 x - \cos^3 x = -1$ ✓

$\rightarrow \sin^3 x - \cos^3 x = -1$ ✓

$\rightarrow \cancel{\sin^3 x - \cos^3 x = 1}$

$\boxed{x = 0 \text{ و } 2\pi \text{ و } \frac{3\pi}{2}}$

جواب ها در بازه ی $[0, 2\pi]$