حسنا محمدی — دوازدهم تجربی

سوال ۱)

الف) $\cos 2x = 3\sin x - 1$

$\rightarrow \cos 2x = 1 - 2\sin^2 x \longrightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$\sin x = \frac{-3 \pm 5}{4}$ $\left\langle \begin{array}{l} \sin x = -2 \ \times \\ \sin x = \frac{1}{2} \ \checkmark \end{array} \right.$

$\rightarrow \sin x = \frac{1}{2}$ $\begin{cases} x \in 2k\pi + \frac{\pi}{6} \\ x \in 2k\pi + \pi - \frac{\pi}{6} \longrightarrow 2k\pi + \frac{5\pi}{6} \end{cases}$

ب) $\cos 2x + \cos x - 2 = 0$

$\cos 2x = 2\cos^2 x - 1 \longrightarrow 2\cos^2 x - 1 + \cos x - 2 = 0$

$\Rightarrow 2\cos^2 x + \cos x - 3 = 0 \longrightarrow \cos x = \frac{-1 \pm 5}{4}$ $\left\langle \begin{array}{l} \cos x = -\frac{3}{2} \ \times \\ \cos x = 1 \ \checkmark \end{array} \right.$

سوال ۲)

الف) $\sin^4 x - \cos^4 x = \sin 4x$

$(\sin^2 x - \cos^2 x)\underbrace{(\sin^2 x + \cos^2 x)}_{1} = -\cos 2x$

$\rightarrow \underbrace{\sin 4x}_{2\sin 2x \cos 2x} = -\cos 2x \longrightarrow 2\sin 2x \cos 2x = -\cos 2x$

$\sin 2x = -\frac{1}{2} \ , \ \cos 2x = 0$

$2x \in 2k\pi + \frac{7\pi}{6} \longrightarrow x \in k\pi + \frac{7\pi}{12}$

$2x \in 2k\pi + \pi - \frac{7\pi}{6} = 2k\pi - \frac{\pi}{6} \longrightarrow x \in k\pi - \frac{\pi}{12}$

$2x \in 2k\pi \pm \frac{\pi}{2} \longrightarrow x \in k\pi \pm \frac{\pi}{4}$

ب) $2\cos^2 x + 2\sin x\cos x = 1$

$1+\cos 2x \quad \sin 2x$

$1 + \cos 2x + \sin 2x = 1 \rightarrow \sin 2x + \cos 2x = 0$

$\cos 2x = -\sin 2x$

$2x \rightarrow 2x \in 2k\pi + \frac{3\pi}{4} \rightarrow x \in k\pi + \frac{3\pi}{8}$

$\rightarrow 2x \in 2k\pi + \frac{7\pi}{4} \rightarrow x \in k\pi + \frac{7\pi}{8}$

سوال ۱۳

الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x}$

$\frac{\cos x}{\sin x}(2\cos x + 1) - \frac{1}{\sin x} = 0 \rightarrow \frac{1}{\sin x}(2\cos^2 x + \cos x - 1) = 0$

$\sin x \neq 0$

$\cos x = \frac{-1 \pm 3}{4}$ → $\cos x = -1$ ✗ ; $\cos x = \frac{1}{2}$

$\cos x = \frac{1}{2} \rightarrow x = 2k\pi \pm \frac{\pi}{3}$ ← کسر قبل قبول ← $\sin x = 0$

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2$

$\underbrace{\sin^2 x + \cos^2 x}_{1} + \sin^2 x - \sin x\cos x = 2$

$\rightarrow \sin^2 x - \sin x\cos x = \sin^2 x + \cos^2 x$

$\rightarrow \cos^2 x + \sin x\cos x = 0 \rightarrow \cos x(\cos x + \sin x) = 0$

$\cos x \neq 0$ ↙ ↘ $\sin x = -\cos x$

$x \rightarrow x \in 2k\pi \pm \frac{\pi}{2}$

$x \in 2k\pi + \frac{3\pi}{4}$

$x \in 2k\pi + \frac{7\pi}{4}$

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سوال ۱۴

الف) $\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4}) = \frac{1}{4}$

$\hookrightarrow \cos(x-\frac{\pi}{4}) = \cos(\frac{\pi}{4}-x)$

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$\rightarrow \sin(x+\frac{\pi}{4})\cos(x+\frac{\pi}{4}) = \frac{1}{4}$

$\frac{1}{2}\sin(2x+\frac{\pi}{2})$

$\rightarrow \cos 2x = \frac{1}{2}$

$2x \in 2k\pi \pm \frac{\pi}{3} \longrightarrow x \in k\pi \pm \frac{\pi}{6}$

ب) $\cos(2x-\frac{\pi}{9}) = -\sin 2x$

$\cos(2x-\frac{\pi}{9}) = \sin(-2x) = \cos(2x+\frac{\pi}{2})$

$2x - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + 2x \longrightarrow \times$

$2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x \longrightarrow x \in \frac{k\pi}{2} - \frac{5\pi}{18}$

سوال ۱۵

$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \rightarrow \sin^2 x + \cos^2 x = 1$

$\rightarrow \frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} = 2\cos 2x + 1$

$\rightarrow 2\cos 2x + 1 = 1 \longrightarrow 2\cos 2x = 0 \longrightarrow \cos 2x = 0$

$x \in k\pi \pm \frac{\pi}{4}$

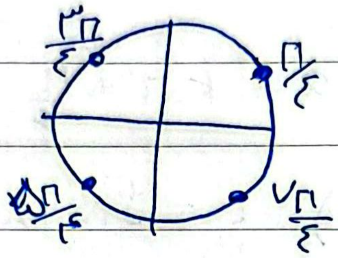
سوال ۶)

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} \qquad \left[-\pi , \frac{3\pi}{2}\right]$$

$$\rightarrow \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \rightarrow \tan\frac{x}{4} = \frac{1}{\tan\frac{x}{4}}$$

$$\tan^2\frac{x}{4} = 1$$

$$\tan\frac{x}{4} = \pm 1$$



$$\frac{x}{4} \in \frac{k\pi}{2} \pm \frac{\pi}{4}$$

$$\left.\begin{array}{l} x \in 2k\pi \pm \pi \\ -\pi \leq x \leq \frac{3\pi}{2} \end{array}\right\} x = -\pi , \pi$$

→ تعداد $= 2$

سوال ۷)

$$\cos^2 x - \sin^2 x \cos 3x = 1 \qquad [0 , 2\pi]$$

$\cos^2 x \rightarrow 1 - \sin^2 x$

$$\rightarrow 1 - \sin^2 x - \sin^2 x \cos 3x = 1 \longrightarrow -\sin^2 x (1 + \cos 3x) = 0$$

$\sin x = 0$, $\cos 3x = -1$

$x \rightarrow x \in k\pi$

$x \rightarrow 3x \in 2k\pi \pm \pi \longrightarrow x \in \frac{2k\pi}{3} \pm \frac{\pi}{3}$

در بازه $[0 , 2\pi]$ $\longrightarrow$ $0 , \pi , 2\pi , \frac{\pi}{3} , \frac{5\pi}{3}$

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سوال ۸)

$\frac{1-\tan x}{1+\tan x} = \tan^3 x$

$\hookrightarrow \frac{\tan\frac{\pi}{4} - \tan x}{1+(\tan\frac{\pi}{4} \times \tan x)} = \tan^3 x$

$\rightarrow \tan(\frac{\pi}{4} - x) = \tan^3 x \rightarrow 3x = k\pi + \frac{\pi}{4} - x$

$\Leftarrow x = k\pi + \frac{\pi}{4}$

$\rightarrow x = k\pi + \frac{\pi}{16}$

سوال ۱۰)

الف) $\underbrace{\sin^2 x - \cos^2 x}_{-\cos 2x} = 1 \quad [0, 2\pi]$

$\rightarrow -\cos 2x = 1 \rightarrow \cos 2x = -1$

$\rightarrow 2x \in \{k\pi \pm \pi\} \rightarrow x \in k\pi \pm \frac{\pi}{2} \rightarrow \boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$

* مقادیر مختلف را امتحان می‌کنیم

ب) $\sin^3 x - \cos^3 x = -1$

$x = 0 \rightarrow 0 - 1 = -1$ ✓ $\quad x = \pi \rightarrow 0 - (-1) \neq -1$ ✗

$x = \frac{\pi}{2} \rightarrow 1 - 0 \neq -1$ ✗ $\quad x = \frac{3\pi}{2} \rightarrow -1 - 0 = -1$ ✓

$x = 2\pi \rightarrow 0 - 1 = -1$ ✓ $\quad 0, \frac{3\pi}{2}, 2\pi$