دریا شکرانیا

۱- الف) $\cos 2x = 3\sin x - 1 \rightarrow \cos 2x = 1 - 2\sin^2 x \rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$\rightarrow \sin x = \frac{-3 \pm \sqrt{25}}{4}$
- $\rightarrow \sin x = -2$ غ ق ق
- $\rightarrow \sin x = \frac{1}{2}$ ق ق $\rightarrow \sin x = \frac{1}{2}$
  - $\rightarrow x \in 2k\pi + \frac{\pi}{6}$
  - $\rightarrow x \in 2k\pi + \pi - \frac{\pi}{6} = 2k\pi + \frac{5\pi}{6}$

ب) $\cos 2x + \cos x - 2 = 0 \rightarrow \cos 2x = 2\cos^2 x - 1 \rightarrow 2\cos^2 x - 1 + \cos x - 2 = 0$

$\rightarrow 2\cos^2 x + \cos x - 3 = 0 \rightarrow \cos x = \frac{-1 \pm \sqrt{25}}{4}$
- $\rightarrow \cos x = -\frac{3}{2}$ غ ق ق
- $\rightarrow \cos x = 1$ ق ق

۲- الف) $\sin^4 x - \cos^4 x = \sin 4x \rightarrow \underbrace{(\sin^2 x + \cos^2 x)}_{1}(\sin^2 x - \cos^2 x) = -\cos 2x$

$\sin 4x = 2\sin 2x \cos 2x = -\cos 2x$
- $\rightarrow \sin 2x = -\frac{1}{2}$
- $\rightarrow \cos 2x = 0$

$\sin 2x = -\frac{1}{2}$
- $\rightarrow 2x \in 2k\pi + \frac{7\pi}{6} \rightarrow x = k\pi + \frac{7\pi}{12}$
- $\rightarrow 2x \in 2k\pi + \pi - \frac{7\pi}{6} \Rightarrow x = k\pi - \frac{\pi}{12}$

$\cos 2x = 0 \rightarrow 2x \in 2k\pi \pm \frac{\pi}{2} \rightarrow x = k\pi \pm \frac{\pi}{4}$

ب) $2\cos^2 x + 2\sin x \cos x = 1$
- $\rightarrow 2\cos^2 x = 1 + \cos 2x$
- $\rightarrow 2\sin x \cos x = \sin 2x$

$\rightarrow 1 + \cos 2x + \sin 2x = 1 \rightarrow \cos 2x + \sin 2x = 0 \rightarrow \cos 2x = -\sin 2x \rightarrow$

- $\rightarrow 2x \in 2k\pi + \frac{3\pi}{4} \rightarrow x = k\pi + \frac{3\pi}{8}$
- $\rightarrow 2x \in 2k\pi + \frac{7\pi}{4} \rightarrow x = k\pi + \frac{7\pi}{8}$

۳.

الف) $\cot x(2\cos x+1)=\frac{1}{\sin x}$

$\frac{\cos x}{\sin x}(2\cos x+1)-\frac{1}{\sin x}=0 \to \frac{1}{\sin x}(2\cos^2 x+\cos x-1)=0$

$\cos x=\frac{-1\pm\sqrt{9}}{4}$

$\to \cos x=-1$ غ ق $\to$ چون $\sin x=0$ می‌شود

$\to \cos x=\frac{1}{2}$ ق ق $\to x\in 2k\pi\pm\frac{\pi}{3}$

ب) $2\sin^2 x-\sin x\cos x+\cos^2 x=2 \to \sin^2 x+\cos^2 x+\sin^2 x-\sin x\cos x=2 \to$

$\to \sin^2 x-\sin x\cos x=\sin^2 x+\cos^2 x \to \cos^2 x+\sin x\cos x=0 \to$

$\to \cos x(\cos x+\sin x)=0$

$\cos x=0 \to x\in 2k\pi\pm\frac{\pi}{2}$

$\sin x=-\cos x \to x=2k\pi+\frac{3\pi}{4}$, $x=2k\pi+\frac{7\pi}{4}$

۴.

الف) $\cos(\frac{\pi}{4}+x)\underbrace{\cos(x-\frac{\pi}{4})}_{\cos(\frac{\pi}{4}-x)}=\frac{1}{4}$

$\sin(x+\frac{\pi}{4})\cos(x+\frac{\pi}{4})=\frac{1}{4} \to \frac{1}{2}\sin(2x+\frac{\pi}{2})=\frac{1}{4} \to$

$\to \sin(2x+\frac{\pi}{2})=\frac{1}{2} \to \cos 2x=\frac{1}{2} \to 2x=2k\pi\pm\frac{\pi}{3} \to x=k\pi\pm\frac{\pi}{6}$

ب) $\cos(2x-\frac{\pi}{9})=-\sin 2x \to \cos(2x-\frac{\pi}{9})=\sin(-2x)=\cos(2x+\frac{\pi}{2})$

$\to 2x-\frac{\pi}{9}=2k\pi+\frac{\pi}{2}+2x$ غ ق و $2x-\frac{\pi}{9}=2k\pi-\frac{\pi}{2}-2x \to$

$\to 4x=2k\pi+\frac{\pi}{9}-\frac{\pi}{2}$

$\to x=\frac{k\pi}{2}-\frac{7\pi}{72}$

$$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \to \frac{\sin 4x + \sin 2x}{\sin 2x} = 1$$ ۵

$$\frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} = 1 \to 2\sin 2x\cos 2x + \sin 2x = \sin 2x \to$$

$$\to 2\sin 2x\cos 2x = 0 \to \sin 4x = 0 \to 4x = k\pi \to x = \frac{k\pi}{4}$$

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2}$$ $[-\pi, \frac{3\pi}{2}]$ ۶

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \to \tan\frac{x}{4} = \frac{1}{\tan\frac{x}{4}} \to \tan^2\frac{x}{4} = 1 \to \tan\frac{x}{4} = \pm 1 \to$$

$$\to \frac{x}{4} = \frac{k\pi}{2} \pm \frac{\pi}{4} \to x \in 2k\pi \pm \pi \xrightarrow{[-\pi, \frac{3\pi}{2}]} \pi, -\pi \to \pi - (-\pi) = 2\pi$$

$$\cos^2 x - \sin^2 x\cos 3x = 1$$ $[0, \pi]$ ۷

$$\cancel{1} - \sin^2 x - \sin^2 x\cos 3x = \cancel{1} \to -\underset{0}{\sin^2 x}(1+\underset{-1}{\cos 3x}) = 0$$

$$-\sin^2 x = 0 \to x \in k\pi$$

$$\cos 3x = -1 \to 3x \in 2k\pi \pm \pi \to x \in \frac{2k\pi}{3} \pm \frac{\pi}{3} \xrightarrow{[0, 2\pi]} 0, \pi, 2\pi, \frac{5\pi}{3}, \frac{\pi}{3}$$

۵ جواب دارد

$$\frac{1-\tan x}{1+\tan x} = \tan 3x \to \frac{\tan\frac{\pi}{4} - \tan x}{1+(\tan\frac{\pi}{4}\tan x)} = \tan 3x \to$$ ۸

$$\to \tan(\frac{\pi}{4} - x) = \tan 3x \to 3x = k\pi + \frac{\pi}{4} - x \to 4x = k\pi + \frac{\pi}{4}$$

$$x = \frac{k\pi}{4} + \frac{\pi}{16}$$

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \rightarrow \sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x} = \sqrt{2}\cos 2x$ و

$|\sin x+\cos x| = \sqrt{2}\cos 2x \rightarrow \sin x = \frac{1}{\sqrt{3}} \rightarrow x = \arcsin\frac{1}{\sqrt{3}}$

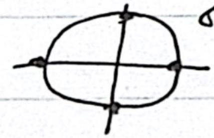
۱۰- $[0, 2\pi]$

الف) $\sin^4 x - \cos^4 x = 1 \rightarrow -\cos 2x = 1 \rightarrow \cos 2x = -1$

$2x = 2k\pi \pm \pi \rightarrow x = k\pi \pm \frac{\pi}{2} \xrightarrow{[0,2\pi]} x = \frac{\pi}{2}$ و $\frac{3\pi}{2}$

ب) $\sin^3 x - \cos^3 x = -1$

• امتحان کردن ۴ نقطه اصلی



$x = 0 \rightarrow -1$ ✓ و $x = \pi \rightarrow +1$ ✗ و $x = 2\pi \rightarrow -1$ ✓ و $x = \frac{\pi}{2} \rightarrow +1$ ✗

$x = \frac{3\pi}{2} \rightarrow -1$ ✓ $\rightarrow x = 0$ و $\frac{3\pi}{2}$ و 2π