

الف) $1 - 2\sin^2 x = \sin^2 x - 1 \Rightarrow 2\sin^2 x + \sin^2 x - 2 = 0 \Rightarrow (\sin x + \frac{1}{2})(\sin x - \frac{1}{2})$

$\sin x = \frac{1}{2} \Rightarrow x = 2k\pi + \frac{\pi}{6}$
 $x = 2k\pi + \frac{5\pi}{6}$

ب) $2\cos^2 x + \cos x - 3 = 0$
 $\cos x = \frac{1}{2} \Rightarrow x = 2k\pi$

الف) $(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = \sin^2 x \Rightarrow \cos^2 x = \sin^2 x \Rightarrow \cos x = \sin(-x)$
 $\sin(\frac{\pi}{2} - x)$

$\frac{\pi}{2} - x = 2k\pi - x \Rightarrow x = k\pi - \frac{\pi}{2}$

$\frac{\pi}{2} - x = 2k\pi + \pi - x \Rightarrow x = k\pi - \frac{\pi}{2}$

ب) $2\cos^2 x + \sin^2 x = 1 \Rightarrow \cos^2 x + \sin^2 x = 0$
 $1 + \cos^2 x$
 $\cos^2 x = \cos(\frac{\pi}{2} + 2k\pi)$
 $x = k\pi - \frac{\pi}{2}$

الف) $\frac{\cos x}{\sin x} (2\cos x + 1) = \frac{1}{\sin x} \Rightarrow 2\cos^2 x + \cos x - 1 = 0$
 $\cos x = \frac{1}{2} \Rightarrow x = 2k\pi \pm \frac{\pi}{3}$

ب) $2\sin^2 x - \sin x \cos x + \cos^2 x = 0$
 $\tan x = 0 \Rightarrow x = k\pi$
 $\tan x = -\frac{1}{2} \Rightarrow x = k\pi - \arctan(\frac{1}{2})$

الف) $\cos(\frac{\pi}{2} + x) \cos(x - \frac{\pi}{2}) = \frac{1}{2} \Rightarrow \frac{1}{2} (\cos^2 x + \sin^2 x) = \frac{1}{2}$
 $\cos^2 x = \frac{1}{2} \Rightarrow x = 2k\pi \pm \frac{\pi}{4}$

ب) $\cos(2x - \frac{\pi}{4}) = \sin(-2x) = \cos(\frac{\pi}{4} + 2x)$
 $2x - \frac{\pi}{4} = 2k\pi - 2x - \frac{\pi}{4}$
 $x = k\pi - \frac{\pi}{4}$

$\frac{\sin^2 x + \sin^2 x}{\sin^2 x} = \cos^2 x + \sin^2 x = 1 \Rightarrow \sin^2 x = \sin^2 x \Rightarrow \sin x = 0$

اگر $\sin x = 0$ برابر صفر است در بعضی از جوابها
 $x = k\pi \Rightarrow x = \frac{k\pi}{1}$
 صفر شود که غلط است چون
 معبر عبارت اول صفر شود!

$$\frac{\sin \frac{x}{2}}{1 + \cos \frac{x}{2}} = \frac{1 + \cos \frac{x}{2}}{\sin \frac{x}{2}} \Rightarrow \frac{\sin \frac{x}{2}}{1 - \cos \frac{x}{2}} = \cos \frac{x}{2} + \frac{1}{\cos \frac{x}{2}}$$

$$\frac{1}{\cos \frac{x}{2}} + \cos \frac{x}{2} = 0 \Rightarrow \cos \frac{x}{2} (\cos \frac{x}{2} + 1) = 0$$

$\cos \frac{x}{2} = 0 \Rightarrow x = 2k\pi + \pi$ (2)

$\cos \frac{x}{2} = -1 \Rightarrow \frac{x}{2} = 2k\pi + \pi \Rightarrow x = 4k\pi + 2\pi$

π و $-\pi$ (یعنی $(-\pi, \frac{\pi}{2})$ بازه) جواب داده باز $\Rightarrow$ $\cos \frac{x}{2} = -1 \Rightarrow \frac{x}{2} = 2k\pi + \pi \Rightarrow x = 4k\pi + 2\pi$

تساوی $= 2\pi$ ✓

$$\cos^2 x - \sin^2 x \cos^2 x = 1 \Rightarrow -\sin^2 x (1 + \cos^2 x) = 0$$

$\sin^2 x = 0 \Rightarrow x = k\pi \Rightarrow x = \frac{k\pi}{2} \Rightarrow 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ (1, 2)

$\cos^2 x = -1 \Rightarrow x = 2k\pi + \pi \Rightarrow x = \frac{2k\pi}{2} + \frac{\pi}{2} \Rightarrow \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = 0, \pi, 2\pi$

جواب در این بازه $\times$ (5)

$$\frac{\cos x - \sin x}{\cos x} = \frac{\cos x \sin x}{\cos x + \sin x} = \tan^2 x$$

توان 2: $1 + \sin^2 x = 2 \cos^2 x \Rightarrow 2 \sin^2 x + \sin^2 x - 1 = 0$

$\sin^2 x = -1 \Rightarrow x = k\pi - \frac{\pi}{2}$ (2)

$\frac{1}{2} \Rightarrow x = k\pi + \frac{\pi}{2}$ (2)

$x = k\pi + \frac{3\pi}{2}$ (2)

جواب ما در این بازه $\frac{\pi}{12}$ و $\frac{5\pi}{12}$ است.

$$\sin^2 x - \cos^2 x = 1 \Rightarrow -\cos^2 x = 1 \Rightarrow \cos(\pi - 2x) = 1$$

$$\pi - 2x = 2k\pi \Rightarrow x = \frac{\pi}{2} - k\pi \Rightarrow \frac{\pi}{2} \text{ و } \frac{3\pi}{2}$$

$$(\sin x - \cos x)(\sin^2 x + \cos^2 x + \sin x \cos x) = -1$$

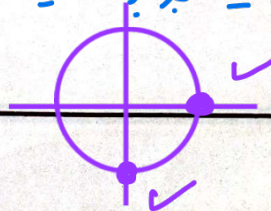
در سوالات به قدر $\sin^2 x + \cos^2 x = 1$ برابر 1 یا -1. فقط کافی است نگاه کنید!

$\sin^2 x - \cos^2 x = 1$

چک کنید!

$$x = 2\pi \text{ یا } 0$$

$$x = \frac{5\pi}{2}$$



$$\frac{y \sin x_n \cos y_n + \sin x_n}{\sin x_n} = \sin x_n + \cos y_n$$

السؤال ٧

$$\frac{\sin x_n (y \cos y_{n+1})}{\sin x_n} = 1 \xrightarrow{\sin x_n \neq 0} y \cos y_{n+1} = 1 \rightarrow \cos y_{n+1} = 1$$

$$y_n = k\pi + \frac{\pi}{4} \rightarrow \boxed{x = k\pi + \frac{\pi}{4}}$$

$$\tan \frac{\pi}{4} = 1 \rightarrow \frac{1 - \tan x_n}{1 + \tan x_n} = \tan \left(\frac{\pi}{4} - x_n \right)$$

السؤال ٨

$$\tan \left(\frac{\pi}{4} - x_n \right) = \tan x_n \rightarrow \frac{\pi}{4} - x_n = k\pi + x_n$$

$$-x_n = k\pi - \frac{\pi}{4} \xrightarrow{-k\pi = k\pi} x_n = k\pi + \frac{\pi}{4}$$

$$\boxed{x = \frac{k\pi}{2} + \frac{\pi}{4}}$$