سیده ثنا بنی طبا — تکلیف شماره ۱۴ — دوازدهم دفتر A - جمعه‌ها

۱

الف) $\cos 2x = 3\sin x - 1 \Rightarrow \cos^2 x - \sin^2 x = 3\sin x - 1$

$1 - 2\sin^2 x = 3\sin x - 1 \Rightarrow 2\sin^2 x + 3\sin x - 2 = 0$ $\quad \sin x = \frac{-3 \pm \sqrt{25}}{4}$ ↗ $\sin x = -2$ غ ق ق ↘ $\sin x = \frac{1}{2}$ ق ق

$\sin x = \sin\frac{\pi}{6} \Rightarrow x = 2k\pi + \frac{\pi}{6}$ و $x = 2k\pi + \frac{5\pi}{6}$ $\quad x = 2k\pi + \frac{\pi}{6}$ و $2k\pi + \frac{5\pi}{6}$

ب) $\cos 2x + \cos x - 2 = 0 \Rightarrow \cos^2 x - \underbrace{\sin^2 x}_{1-\cos^2 x} + \cos x - 2 = 0 \Rightarrow 2\cos^2 x + \cos x - 3 = 0$ $\quad a+b+c=0$

$\rightarrow \cos x = 1$ ق ق

$\cos x = -\frac{3}{2}$ غ ق ق

$\cos x = 1 \quad \cos x = \cos 2k\pi \Rightarrow x = 2k\pi$

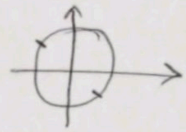
۲

الف) $\underbrace{(\cos^2 x + \sin^2 x)}_{1}\underbrace{(\sin^2 x - \cos^2 x)}_{-\cos 2x} = \sin 4x \Rightarrow -\cos 2x = 2\sin 2x \cos 2x \Rightarrow \sin 2x = -\frac{1}{2}$

$2x = 2k\pi - \frac{\pi}{6}$ و $2x = 2k\pi - \frac{5\pi}{6} \Rightarrow x = k\pi - \frac{\pi}{12}$ و $k\pi - \frac{5\pi}{12}$

ب) $2\cos^2 x + 2\sin x\cos x = 1 \Rightarrow 2\cos^2 x + \sin 2x - \sin^2 x - \cos^2 x \Rightarrow \underbrace{\cos^2 x - \sin^2 x}_{\cos 2x} + \sin 2x = 0$

$\rightarrow \sin^2 x + \cos^2 x$



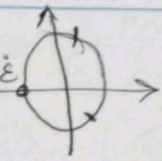
$\cos 2x + \sin 2x = 0 \Rightarrow \cos 2x = -\sin 2x$ $\quad 2x = k\pi - \frac{\pi}{4} \Rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$

۳

الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x} \Rightarrow \frac{\cos x}{\sin x} \times (2\cos x + 1) = \frac{1}{\sin x}$

$2\cos^2 x + \cos x - 1 = 0 \quad a + c = b \Rightarrow \cos x = -1$ غ ق ق $\quad \cos x = \frac{1}{2}$

sin صفر است و تعریف نمی‌شود!



$\cos x = \frac{1}{2} \Rightarrow x = 2k\pi \pm \frac{\pi}{3}$

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x - \underbrace{2}_{-2\sin^2 x - 2\cos^2 x} = 0$ $\quad -\cos^2 x - \sin x\cos x = 0 \Rightarrow \cos x(\cos x + \sin x) = 0$

$\rightarrow \cos x = 0 \quad k\pi + \frac{\pi}{2}$

$\rightarrow \cos x = -\sin x \quad k\pi - \frac{\pi}{4}$

$x = k\pi + \frac{\pi}{2}$ و $k\pi - \frac{\pi}{4}$

۴

الف) $\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \frac{1}{4} \Rightarrow \cos(\frac{\pi}{4} + x)\cos(\frac{\pi}{4} - x) = \frac{1}{4}$ $\quad \cos\alpha = \cos(-\alpha)$

$(\frac{\pi}{4} + x + \frac{\pi}{4} - x) = \frac{\pi}{2}$ (دو زاویه متمم هستند) $\Rightarrow \cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{4} + x) = \frac{1}{4} \Rightarrow \frac{1}{2}\sin(\frac{\pi}{2} + 2x) = \frac{1}{4}$

$\frac{1}{2}\cos 2x = \frac{1}{4} \Rightarrow \cos 2x = \frac{1}{2}$ $\quad 2x = 2k\pi + \frac{\pi}{3} \rightarrow x = k\pi + \frac{\pi}{6}$, $\quad 2x = 2k\pi - \frac{\pi}{3} \rightarrow x = k\pi - \frac{\pi}{6}$ $\quad \Big\} \; x = k\pi \pm \frac{\pi}{6}$

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x \Rightarrow \cos(2x - \frac{\pi}{9}) = \sin(-2x) \Rightarrow \cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x)$

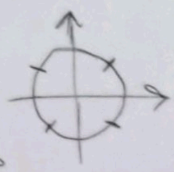
$\rightarrow 2x - \frac{\pi}{9} = \frac{\pi}{2} + 2x$ غ ق ق

$\rightarrow 2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x \Rightarrow 4x = 2k\pi - \frac{\pi}{2} + \frac{\pi}{9} \Rightarrow 4x = 2k\pi - \frac{7\pi}{18} \Rightarrow x = \frac{k\pi}{2} - \frac{7\pi}{72}$

۵

$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \Rightarrow \frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x$

$2\cos 2x + 1 = \sin^2 x + \cos^2 x \Rightarrow \cos 2x = 0$ $\quad 2x = k\pi + \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$



هیچکدام از جواب‌های به دست آمده $\sin 2x$ را صفر نمی‌کند و تعریف شده هستند.

⑥

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \Rightarrow \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \Rightarrow \sin^2\frac{x}{2} = 1+\cos^2\frac{x}{2}+2\cos\frac{x}{2}$$

$$\cancel{1}-\cos^2\frac{x}{2} = \cancel{1}+\cos^2\frac{x}{2}+2\cos\frac{x}{2} \Rightarrow 2\cos^2\frac{x}{2}+2\cos\frac{x}{2}=0 \Rightarrow \cos\frac{x}{2}\left(\cos\frac{x}{2}+1\right)=0$$

$\rightarrow \cos\frac{x}{2}=0 \quad \frac{x}{2}=k\pi+\frac{\pi}{2} \Rightarrow x=2k\pi+\pi \rightarrow k=0\rightarrow\pi \quad k=-1\rightarrow-\pi$

$\hookrightarrow \cos\frac{x}{2}=-1 \quad \frac{x}{2}=2k\pi+\pi \Rightarrow x=4k\pi+2\pi \rightarrow$ در این بازه جواب ندارد $\quad |\pi-(-\pi)| = \underline{\underline{2\pi}}$

⑦

$\cos^2 x-\sin^2 x\cos 3x=1 \Rightarrow -\sin^2 x\cos 3x = \underbrace{1-\cos^2 x}_{\sin^2 x} \Rightarrow \cos 3x=-1$

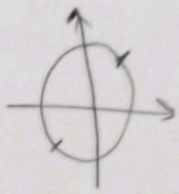
$3x=2k\pi+\pi \Rightarrow x=\frac{2k\pi}{3}+\frac{\pi}{3} \qquad k=0\rightarrow\frac{\pi}{3},\ k=1\rightarrow\pi,\ k=2\rightarrow\frac{5\pi}{3}$ دارای ۵ جواب!

$\sin x=0 \quad x=k\pi \quad k=0\rightarrow 0 \quad k=1\rightarrow\pi \quad k=2\rightarrow 2\pi$

$\Rightarrow x=\frac{\pi}{3},\ \pi,\ \frac{5\pi}{3},\ 0,\ 2\pi$

⑧

$$\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}} = \frac{\sin 3x}{\cos 3x} \Rightarrow \frac{\frac{\cos x-\sin x}{\cos x}}{\frac{\cos x+\sin x}{\cos x}} = \frac{\sin 3x}{\cos 3x} \Rightarrow \frac{\cos x-\sin x}{\cos x+\sin x} = \frac{\sin 3x}{\cos 3x}$$



$\sin 3x\cos x+\sin 3x\sin x = \cos 3x\cos x-\cos 3x\sin x$

$\sin 3x\cos x+\cos 3x\sin x = \cos 3x\cos x-\sin 3x\sin x \Rightarrow \sin 4x=\cos 4x$

$4x=k\pi+\frac{\pi}{4} \Rightarrow x=\frac{k\pi}{4}+\frac{\pi}{16}$

⑨

$\sqrt{1+\sin 2x}=\sqrt{2}\cos 2x \Rightarrow \sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x}=\sqrt{2}\cos 2x$

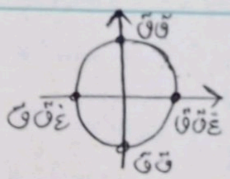
$|\sin x+\cos x|=\sqrt{2}\cos 2x \rightarrow \sqrt{2}\sin\left(\frac{\pi}{4}+x\right)=\sqrt{2}\cos 2x \Rightarrow \sin\left(\frac{\pi}{4}+x\right)=\sin\left(\frac{\pi}{2}-2x\right)$

$x=\frac{2k\pi}{3}+\frac{\pi}{12}$

$\rightarrow -\sqrt{2}\sin\left(\frac{\pi}{4}+x\right)=\sqrt{2}\cos 2x \Rightarrow \sin\left(\frac{-\pi}{4}-x\right)=\sin\left(\frac{\pi}{2}-2x\right) \Rightarrow x=2k\pi+\frac{3\pi}{4} \qquad \underbrace{\frac{3\pi}{4},\ \frac{9\pi}{12},\ \frac{\pi}{12}}_{\text{دارای ۳ جواب}}$

⑩

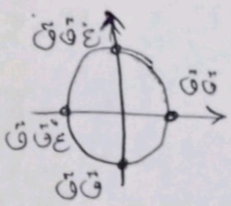
الف) $\sin^4 x-\cos^4 x=1$



$\Rightarrow x=\frac{\pi}{2},\ \frac{3\pi}{2}$

sin و cos به توان یکسان رسیده‌اند و جواب ۱ شده است. پس باید ۴ نقطه‌ی اصلی را چک کنیم.

ب) $\cos^3 x-\sin^3 x=1$



$x=0,\ \frac{3\pi}{2},\ 2\pi$