۱۷,۷۵

نفس زنده دل

$1+\tan^2 x = 1/\cos^2 x \Rightarrow 1/\cos^2 x = 8\cos x \Rightarrow 8\cos^3 x = 1 \Rightarrow \cos^3 x = 1/8$ -۱

$\cos x = 1/2 \Rightarrow 2k\pi \pm \pi/3 \rightarrow$ جواب ۲

۲

$$\sin^2 x = \frac{1-\cos ۲x}{۲}$$

$$۵\left(\frac{1-\cos ۲x}{۲}\right) + ۲\cos(۳x) = -۲ \overset{\times ۲}{\Rightarrow} ۵ - ۵\cos ۲x + ۴\cos ۳x = -۴$$

$$-۵\cos ۲x + ۴\cos ۳x = -۹ \longrightarrow -۱۰\cos^2 x + ۵ + ۱۶\cos^3 x - ۱۲\cos x = -۹$$

$$\cos ۲x \to (۲\cos^2 x - 1), \quad \cos ۳x \to (۴\cos^3 x - ۳\cos x)$$

$$\Rightarrow -۱۰ - ۱۴ + ۱۶ - ۱۲ \Rightarrow (\cos x + 1)$$

$$\begin{array}{l|l} +۱۶\cos^3 x - ۱۰\cos^2 x - ۱۲\cos x + ۱۴ & \cos x + 1 \\ \hline ۱۶\cos^3 x + ۱۶\cos^2 x & ۱۶\cos^2 x - ۲۶\cos x + ۱۴ \\ \hline -۲۶\cos^2 x - ۱۲\cos x & \\ -۲۶\cos^2 x - ۲۶\cos x & \\ \hline ۱۴\cos x + ۱۴ & \\ ۱۴\cos x + ۱۴ & \\ \hline ۰ & \end{array}$$

$۱۶\cos^2 x - ۲۶\cos x + ۱۴ \downarrow$ ریشه ندارد

۲

$$\Rightarrow \cos x = -1 \qquad x = \pm\pi$$

جواب ۲ $\longrightarrow$

۱,۵ ۳

$$2\sin(x)\cos(2x)+\sin x=1$$

$$\sin x\,(2\cos 2x+1)=1 \Rightarrow \sin x\,(2\cos^2 x-2\sin^2 x+1)=1 \Rightarrow (3-4\sin^2 x)\sin x=1$$

$\cos 2x \to \cos^2 x-\sin^2 x$; $\quad 2\cos^2x - 2\sin^2 x \to 2-4\sin^2 x$

$$3\sin x-4\sin^3 x-1=0 \qquad \sin x=-1 \to \sin x+1$$

$$\begin{array}{l|l} -4\sin^3 x+3\sin x-1 & \sin x+1 \\ -4\sin^3 x-4\sin^2 x & -4\sin^2 x+4\sin x-1 \\ \hline 4\sin^2 x+3\sin x & \to -(4\sin^2 x-4\sin x+1) \\ 4\sin^2 x+4\sin x & \\ \hline -\sin x-1 & \\ -\sin x-1 & \\ \hline 0 & \end{array}$$

$$\Rightarrow (\sin x+1)(2\sin x-1)^2=0$$

$\sin x=-1 \longrightarrow 2k\pi+\cancel{\frac{3\pi}{2}}$

$2\sin x-1=0 \Rightarrow \sin x=\frac{1}{2} \longrightarrow \left\{2k\pi+\cancel{\frac{\pi}{6}},\ 2k\pi+\cancel{\frac{5\pi}{6}}\right\}$

$\cos 2x = 1-2\sin^2 x$ (فرمول طلایی)

$2\sin x(1-2\sin^2 x)+\sin x=1 \longrightarrow 3\sin x-4\sin^3 x=1$ ۳ ۴ ۳

$\sin 3x=1 \longrightarrow 3x=2k\pi+\frac{\pi}{2} \longrightarrow x=\frac{2k\pi}{3}+\frac{\pi}{6} \xrightarrow{0\le x\le 2\pi}$

$k=0 \to x=\frac{\pi}{6}$, $k=1 \to x=\frac{5\pi}{6}$, $k=2 \to x=\frac{9\pi}{6}$

$S=\frac{\pi}{6}+\frac{5\pi}{6}+\frac{9\pi}{6}=\frac{15\pi}{6}=\boxed{\frac{5\pi}{2}}$

مجموع جواب ها؟

$$\sin\left(\frac{\pi}{2}+2x\right) = \cos 2x$$
$$\cos\left(\frac{\pi}{2}+4x\right) = -\sin 4x$$

$$\Rightarrow \frac{1}{\underbrace{\cos 2x}_{\neq 0}} + \frac{1}{\underbrace{-\sin 4x}_{\neq 0}} = 0 \Rightarrow \frac{1}{\cos 2x} = \frac{1}{\sin 4x}$$

-4

2

$$\cos 2x = \sin 4x \Rightarrow \cancel{\cos 2x} = 2\sin 2x \cancel{\cos 2x} \Rightarrow \frac{1}{2} = \sin 2x$$

$$2x = 2k\pi + \frac{\pi}{6}, \quad 2x = 2k\pi + \frac{5\pi}{6}$$

$$\Rightarrow x = k\pi + \frac{\pi}{12}, \quad k\pi + \frac{5\pi}{12} \Rightarrow \alpha = \frac{4\pi}{12} = \frac{\pi}{3} \Rightarrow \tan\frac{2\pi}{3} = -\sqrt{3}$$

۵-

$\cos(x+\frac{\pi}{4}) = \cos x \times \cos\frac{\pi}{4} - \sin x \times \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}}$

$\Rightarrow \cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \frac{\times\sqrt{3}}{\times\sqrt{3}} = \frac{\sqrt{6}}{3} \Rightarrow \frac{\sqrt{6}}{3}m - 2\sqrt{6}\sin 2x = \sqrt{6} \Rightarrow \frac{m}{3} - 2\sin 2x = 1$ ۲

$\Rightarrow \frac{m}{3} - 2(2\sin x\cos x) = 1 \Rightarrow m = 3 + 12\sin x\cos x$ $\qquad (\cos x - \sin x)^2 = 1 - 2\sin x\cos x = \frac{6}{9}$

$m - 9 \times 2\sin x\cos x = 3$ $\qquad \Rightarrow 2\sin x\cos x = \frac{1}{3}$

$\Rightarrow m = 3 + 9 \times \frac{1}{3} = 6$

۶-

$\sin(x+\frac{\pi}{6})\cos(x-\frac{\pi}{3}) = 1$ $\qquad \sin(x+\frac{\pi}{6}) = 1$ و $\cos(x-\frac{\pi}{3}) = 1$

$\sin(x+\frac{\pi}{6}) = -1$ و $\cos(x-\frac{\pi}{3}) = -1$ ۱/۷۵

$x+\frac{\pi}{6} = 2k\pi + \frac{\pi}{2} \Rightarrow x = 2k\pi + \frac{3\pi - \pi}{6} = 2k\pi + \frac{\pi}{3}$ $\qquad x - \frac{\pi}{3} = 2k\pi \Rightarrow x = 2k\pi + \frac{\pi}{3}$

$x+\frac{\pi}{6} = 2k\pi - \frac{\pi}{2} \Rightarrow x = 2k\pi - \frac{3\pi - \pi}{6} = 2k\pi - \frac{2\pi}{3}$ یا $2k\pi + \frac{7\pi}{3}$ ($k=1$) $\qquad x - \frac{\pi}{3} = 2k\pi + \pi \Rightarrow x = 2k\pi + \frac{4\pi}{3}$

$\to 2k\pi + \frac{7\pi}{3}$ و $2k\pi + \frac{4\pi}{3} \to \frac{4\pi}{3}, \frac{7\pi}{3}$ جواب نداره $\Rightarrow k=0 \to \frac{\pi}{3}$ فقط جواب دارد

چرا ؟!!

$\cos(x - \frac{\pi}{3}) = \cos(\frac{\pi}{3} - x)$

$\frac{\pi}{3} - x + x + \frac{\pi}{6} = \frac{\pi}{2} \longrightarrow \cos(\frac{\pi}{3} - x) = \sin(x + \frac{\pi}{6})$

$\sin^2(x+\frac{\pi}{6}) = 1 \longrightarrow \sin(x+\frac{\pi}{6}) = \pm 1 \longrightarrow x + \frac{\pi}{6} = k\pi + \frac{\pi}{2}$

$x = k\pi + \frac{\pi}{3} \xrightarrow{0 \le x \le 2\pi} k=0 \to x = \frac{\pi}{3}$ ، $k=1$ ، $x = \frac{4\pi}{3}$

معادله ۲ جواب در بازه دارد

۷-

$$\sqrt{۲} = \cos x \sqrt{۳} + \sin x \xrightarrow{\times \frac{۱}{۲}} \frac{\sqrt{۲}}{۲} = \underbrace{\cos x \cdot \frac{\sqrt{۳}}{۲} + \frac{۱}{۲} \sin x}_{\sin\left(x + \frac{\pi}{۶}\right)} \Rightarrow x + \frac{\pi}{۶} = ۲k\pi + \frac{\pi}{۴}$$

$$x + \frac{\pi}{۶} = ۲k\pi + \frac{۳\pi}{۴}$$

$$x = ۲k\pi + \frac{۳\pi}{۱۲} - \frac{۴\pi}{۱۲} = ۲k\pi - \frac{\pi}{۱۲} \Rightarrow k = 0 \longrightarrow -\frac{\pi}{۱۲} \,,\, \frac{۵\pi}{۱۲} \qquad k = \cancel{۱}$$

$$x = ۲k\pi + \frac{۹\pi}{۱۲} - \frac{۲\pi}{۱۲} = ۲k\pi + \frac{۵\pi}{۱۲} \qquad k = ۱ \longrightarrow \frac{۲۳\pi}{۱۲} \qquad \Rightarrow \frac{۲۸\pi}{۱۲} - \frac{\pi}{۱۲} = \frac{۲۷\pi}{۱۲} = \frac{۳\pi}{۱۲} = \frac{\pi}{۴}$$

$$\longrightarrow \frac{۹\pi}{۴}$$

۸-

$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x \Rightarrow -4\sin x \cos x = 1$ $\qquad 2\sin x \cos x = \sin 2x$

$\Rightarrow -2(2\sin x \cos x) = 1 \Rightarrow \sin 2x = -\frac{1}{2}$

$2x = 2k\pi - \frac{\pi}{6}$; $x = k\pi - \frac{\pi}{12}$

$2x = 2k\pi - \frac{5\pi}{6}$; $x = k\pi - \frac{5\pi}{12}$

$k=1 \rightarrow \frac{11\pi}{12}, \frac{7\pi}{12}$ $\Rightarrow \frac{11\pi + 7\pi + 19\pi + 23\pi}{12} = \frac{60\pi}{12} = \frac{10\pi}{2} = 5\pi$

$k=2 \rightarrow \frac{23\pi}{12}, \frac{19\pi}{12}$

۹-

$1+\cos 2\alpha = 1 + \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha$

$1+\cos 4\alpha = 1 + \cos^2 2\alpha - \sin^2 2\alpha = 2\cos^2 2\alpha = 2(\cos^2\alpha - \sin^2\alpha)^2$

$\downarrow$ $1 - 2\cos^2\alpha \sin^2\alpha$

$1+\cos 8\alpha = 1 + \cos^2 4\alpha - \sin^2 4\alpha = 2\cos^2 4\alpha = 2(\cos^2 2\alpha - \sin^2 2\alpha)^2 = 2 - 4\cos^2 2\alpha \sin^2 2\alpha$

$\downarrow$ $1 - 2\cos^2 2\alpha \sin^2 2\alpha$

$2\cos 2\alpha \sin 2\alpha = \sin 4\alpha \Rightarrow 4\cos^2 2\alpha \sin^2 2\alpha = \sin^2 4\alpha$

$\Rightarrow (1+\cos 2\alpha)(2\cos^2 2\alpha)(2 - 4\cos^2 2\alpha \sin^2 2\alpha) = \frac{1}{8}$

$2\cos^2 2\alpha = 1 + \cos^2 4\alpha$ $\qquad 1 + \cos^2 4\alpha$

$\Rightarrow (1+\cos^2 4\alpha)^2 (1 + \cos 2\alpha)$

$\frac{1+\cos 4\alpha}{2} = \cos^2 2\alpha$

-1.

$$\tan 3x = \frac{\sin 3x}{\cos 3x} = \frac{3\sin x - 4\sin^3 x}{4\cos^3 x - 3\cos x} \Rightarrow \frac{\sin x\,(3 - 4\sin^2 x)}{\cos x\,(4\cos^2 x - 3)} \cdot \frac{\sin x}{\cos x} = 1$$

$$\frac{\cos^2 x}{\sin^2 x} = \frac{3 - 4\sin^2 x}{4\cos^2 x - 3} \Rightarrow \frac{\cos^2 x}{\sin^2 x} = \frac{3\cos^2 x - \sin^2 x}{3\sin^2 x - \cos^2 x} \Rightarrow 3\sin^2 x\cos^2 x - \cos^4 x = 3\sin^2 x\cos^2 x - \sin^4 x$$

$$\cos^4 x - \sin^4 x = 0 \Rightarrow (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) = 0 \Rightarrow \cos 2x = 0 \quad 2x = 2k\pi \pm \frac{\pi}{2}$$

$$\Rightarrow x = k\pi \pm \frac{\pi}{4} \to k=1 \to \frac{5\pi}{4},\quad k=2 \to \frac{7\pi}{4} \Rightarrow \frac{7\pi}{4} + \frac{5\pi}{4} = \frac{12\pi}{4} = 3$$

1,5

$$an\ 3x = \frac{1}{\tan x} = \cot x \to \tan 3x = \tan\left(\frac{\pi}{2} - x\right)$$

$$3x = k\pi + \frac{\pi}{2} - x \to 4x = k\pi + \frac{\pi}{2} \to x = \frac{k\pi}{4} + \frac{\pi}{8}$$

k	4	5	6	7
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$$S = \left(\frac{9 + 11 + 13 + 15}{8}\right)\pi = \frac{48\pi}{8} = \boxed{6\pi}$$

$1+\cos 2x = 2\cos^2 x$ ★

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \rightarrow 8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\cos 2\alpha\cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha \ ★}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\cos 4\alpha)}_{\sin 8\alpha} = \frac{\pm\sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \rightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \rightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \rightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \rightarrow x = \frac{7\pi}{9}$$

$$\rightarrow \boxed{\max A = \frac{8\pi}{9}}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!