نوشین زنده‌دل

$1+\tan^2 x = \frac{1}{\cos^2 x} \Rightarrow \frac{1}{\cos^2 x} = 8\cos x \Rightarrow 8\cos^3 x = 1 \Rightarrow \cos^3 x = \frac{1}{8}$ ۱-

$\cos x = \frac{1}{2} \Rightarrow 2k\pi \pm \frac{\pi}{3} \longrightarrow$ ۲ جواب

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$\sin^2 x = \frac{1 - \cos 2x}{2}$ $\quad$ $5\left(\frac{1-\cos 2x}{2}\right) + 2\cos(3x) = -2 \overset{\times 2}{\Rightarrow} 5 - 5\cos 2x + 4\cos 3x = -4$

$-5\cos 2x + 4\cos 3x = -9 \longrightarrow -10\cos^2 x + 5 + 16\cos^3 x - 12\cos x = -9$

$(2\cos^2 x - 1)$ $\quad$ $(4\cos^3 x - 3\cos x)$

$\Rightarrow -10 - 14 + 16 - 12 \Rightarrow (\cos x + 1)$

$$\begin{array}{l|l} 16\cos^3 x - 10\cos^2 x - 12\cos x + 14 & \cos x + 1 \\ \underline{16\cos^3 x + 16\cos^2 x} & 16\cos^2 x - 26\cos x + 14 \\ -26\cos^2 x - 12\cos x & \\ \underline{-26\cos^2 x - 26\cos x} & \\ 14\cos x + 14 & \\ \underline{14\cos x + 14} & \\ 0 & \end{array}$$

ریشه ندارد

$\Rightarrow \cos x = -1 \quad x = \pm\pi$

→ دوم

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$$2\sin(x)\cos(2x)+\sin x=1$$

$$\sin x\,(2\cos 2x+1)=1 \quad \Rightarrow \quad \sin x\,(2\cos^2 x-2\sin^2 x+1)=1 \quad \Rightarrow \quad (3-4\sin^2 x)\sin x=1$$

$$\cos 2x \to \cos^2 x-\sin^2 x \qquad\qquad 2\cos^2 x \to 2-2\sin^2 x$$

$$3\sin x-4\sin^3 x-1=0 \qquad \sin x=-1 \;\longrightarrow\; \sin x+1$$

$$\begin{array}{l|l} -4\sin^3 x+3\sin x-1 & \sin x+1 \\ -4\sin^3 x-4\sin^2 x & -4\sin^2 x+4\sin x-1 \\ \hline 4\sin^2 x+3\sin x & \\ 4\sin^2 x+4\sin x & \\ \hline -\sin x-1 & \\ -\sin x-1 & \\ \hline 0 & \end{array}$$

$$-4\sin^2 x+4\sin x-1 \to -(4\sin^2 x-4\sin x+1)$$

$$\Rightarrow (\sin x+1)(2\sin x-1)^2=0$$

$$\sin x=-1 \longrightarrow 2k\pi+\frac{3\pi}{2}$$

$$2\sin x-1=0 \Rightarrow \sin x=\frac{1}{2} \longrightarrow \left\{2k\pi+\frac{\pi}{6},\; 2k\pi+\frac{5\pi}{6}\right\}$$

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$$\sin\left(\frac{\pi}{2}+2x\right)=\cos 2x \qquad \cos\left(\frac{\pi}{2}+4x\right)=-\sin 4x$$

$$\Rightarrow \frac{1}{\underbrace{\cos 2x}_{\neq 0}}+\frac{1}{\underbrace{-\sin 4x}_{\neq 0}}=0 \Rightarrow \frac{1}{\cos 2x}=\frac{1}{\sin 4x}$$

$$\cos 2x=\sin 4x \Rightarrow \cos 2x = 2\sin 2x \cancel{\cos 2x} \Rightarrow \frac{1}{2}=\sin 2x \qquad 2x=2k\pi+\frac{\pi}{6}\ ,\ 2x=2k\pi+\frac{5\pi}{6}$$

$$\Rightarrow x=k\pi+\frac{\pi}{12}\ ,\ k\pi+\frac{5\pi}{12} \Rightarrow \alpha = \frac{4\pi}{12}=\frac{\pi}{3} \Rightarrow \tan\frac{2\pi}{3}=-\sqrt{3}$$

$$\cos\left(x+\frac{\pi}{4}\right) = \cos x \cdot \cos\frac{\pi}{4} - \sin x \cdot \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}}$$ ۵-

$$\Rightarrow \cos x - \sin x = \frac{\sqrt{2}^{\cdot\sqrt{3}}}{\sqrt{3}\cdot\sqrt{3}} = \frac{\sqrt{6}}{3} \Rightarrow \frac{\sqrt{6}}{3}m - 2\sqrt{6}\sin 2x = \sqrt{6} \Rightarrow \frac{m}{3} - 2\sin 2x = 1$$

$$\Rightarrow \frac{m}{3} - 3(2\sin x\cos x) = 1 \Rightarrow m = 3 + 18\sin x\cos x$$

$$m - 9\cdot 2\sin x\cos x = 3$$

$$(\cos x - \sin x)^2 = 1 - 2\sin x\cos x = \frac{6}{9}$$

$$\Rightarrow 2\sin x\cos x = \frac{1}{3}$$

$$\Rightarrow m = 3 + 9\cdot\frac{1}{3} = 6$$

$$\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{3}\right) = 1 \qquad \sin\left(x+\frac{\pi}{6}\right) = 1 \text{ و } \cos\left(x-\frac{\pi}{3}\right) = 1$$ ۶-

$$\sin\left(x+\frac{\pi}{6}\right) = -1 \text{ و } \cos\left(x-\frac{\pi}{3}\right) = -1$$

$$x+\frac{\pi}{6} = 2k\pi+\frac{\pi}{2} \Rightarrow x = 2k\pi + \frac{3\pi-\pi}{6} = 2k\pi+\frac{\pi}{3} \qquad x-\frac{\pi}{3} = 2k\pi \Rightarrow x = 2k\pi+\frac{\pi}{3}$$

$$x+\frac{\pi}{6} = 2k\pi-\frac{\pi}{2} \Rightarrow x = 2k\pi - \frac{3\pi-\pi}{6} = 2k\pi - \frac{2\pi}{3} \xrightarrow{k=1} 2k\pi+\frac{7\pi}{3} \qquad x-\frac{\pi}{3} = 2k\pi+\pi \Rightarrow x = 2k\pi+\frac{4\pi}{3}$$

$$\rightarrow 2k\pi+\frac{7\pi}{3} \text{ و } 2k\pi+\frac{4\pi}{3} \rightarrow \frac{4\pi}{3} \neq \frac{7\pi}{3} \Rightarrow k=0 \rightarrow \frac{\pi}{3}$$ فقط جواب دارد

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$$\sqrt{2}=\cos x\sqrt{3}+\sin x \xrightarrow{\div 2} \frac{\sqrt{2}}{2}=\underbrace{\cos x\cdot\frac{\sqrt{3}}{2}+\frac{1}{2}\sin x}_{\sin\left(x+\frac{\pi}{3}\right)} \Rightarrow \begin{cases} x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \\ x+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4} \end{cases}$$

$$x=2k\pi+\frac{3\pi}{12}-\frac{4\pi}{12}=2k\pi-\frac{\pi}{12} \Rightarrow k=0 \rightarrow -\frac{\pi}{12}\ ,\ \frac{5\pi}{12} \qquad k=\cancel{\ }$$

$$x=2k\pi+\frac{9\pi}{12}-\frac{4\pi}{12}=2k\pi+\frac{5\pi}{12} \qquad k=1 \rightarrow \frac{29\pi}{12}$$

$$\Rightarrow \frac{28\pi}{12}-\frac{\pi}{12}=\frac{27\pi}{12}=\frac{3\pi}{12}=\frac{\pi}{4}$$

$$\rightarrow \frac{9\pi}{4}$$

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$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x \Rightarrow -4\sin x \cos x = 1$ $\qquad 2\sin x \cos x = \sin 2x$

$\Rightarrow -2(2\sin x \cos x) = 1 \Rightarrow \sin 2x = -\frac{1}{2}$ $\qquad \left.\begin{array}{l} 2x = 2k\pi - \frac{\pi}{6} \\ 2x = 2k\pi - \frac{5\pi}{6} \end{array}\right\} \begin{array}{l} x = k\pi - \frac{\pi}{12} \\ x = k\pi - \frac{5\pi}{12} \end{array}$

$k = 1 \longrightarrow \frac{11\pi}{12}, \frac{7\pi}{12}$ $\qquad \Rightarrow \frac{11\pi + 7\pi + 19\pi + 23\pi}{12} = \frac{60\pi}{12} = \frac{10\pi}{2} = 5\pi$

$k = 2 \longrightarrow \frac{23\pi}{12}, \frac{19\pi}{12}$

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$1 + \cos 2\alpha = 1 + \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha$

$1 + \cos 4\alpha = 1 + \cos^2 2\alpha - \sin^2 2\alpha = 2\cos^2 2\alpha = 2(\cos^2\alpha - \sin^2\alpha)^2$

$\downarrow$ $1 - 4\cos^2\alpha \sin^2\alpha$

$1 + \cos 8\alpha = 1 + \cos^2 4\alpha - \sin^2 4\alpha = 2\cos^2 4\alpha = 2(\cos^2 2\alpha - \sin^2 2\alpha)^2 = 2 - 4\cos^2 2\alpha \sin^2 2\alpha$

$\downarrow$ $1 - 2\cos^2 2\alpha \sin^2 2\alpha$

$2\cos 2\alpha \sin 2\alpha = \sin 4\alpha \Rightarrow 4\cos^2 2\alpha \sin^2 2\alpha = \sin^2 4\alpha$

$\Rightarrow (1 + \cos 2\alpha)(2\cos^2 2\alpha)(2 - 4\cos^2 2\alpha \sin^2 2\alpha) = \frac{1}{8}$

$2\cos^2 2\alpha = 1 + \cos^2 4\alpha$ $\qquad 2 - 4\cos^2 2\alpha \sin^2 2\alpha \to 1 + \cos^2 4\alpha$

$\Rightarrow (1 + \cos^2 4\alpha)^2 (1 + \cos 2\alpha)$

$\downarrow$

$\cancel{1+\cos^2 2\alpha}$ $\quad \frac{1 + \cos 4\alpha}{2} = \cos^2 2\alpha$

1.

$$\tan 3x = \frac{\sin 3x}{\cos 3x} = \frac{3\sin x - 4\sin^3 x}{4\cos^3 x - 3\cos x} \Rightarrow \frac{\sin x\,(3 - 4\sin^2 x)}{\cos x\,(4\cos^2 x - 3)} \cdot \frac{\sin x}{\cos x} = 1$$

$$\frac{\cos^2 x}{\sin^2 x} = \frac{3 - 4\sin^2 x}{4\cos^2 x - 3} \Rightarrow \frac{\cos^2 x}{\sin^2 x} = \frac{3\cos^2 x - \sin^2 x}{3\sin^2 x - \cos^2 x} \Rightarrow 3\sin^2 x\cos^2 x - \cos^4 x = 3\sin^2 x\cos^2 x - \sin^4 x$$

$$\cos^4 x - \sin^4 x = 0 \Rightarrow (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) = 0 \Rightarrow \cos 2x = 0 \quad 2x = 2k\pi \pm \pi/2$$

$$\Rightarrow x = k\pi \pm \pi/4 \rightarrow k=1 \rightarrow 5\pi/4, \quad k=2 \rightarrow 7\pi/4 \Rightarrow 7\pi/4 + 5\pi/4 = 12\pi/4 = 3$$