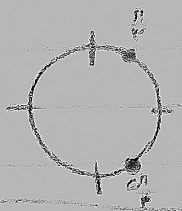
سوال ① در بازه‌ی $[0, 2\pi]$

$$8\cos x - 4\tan^2 x = 1 \Rightarrow 8\cos x = 1 + \underbrace{4\tan^2 x}_{\frac{4}{\cos^2 x}}$$

$$\Rightarrow 8\cos x = \frac{4}{\cos^2 x} \Rightarrow 8\cos^3 x = 4 \Rightarrow \cos^3 x = \frac{1}{8} \Rightarrow \cos x = \frac{1}{2}$$



$$\Rightarrow \boxed{x = 2\pi \pm \frac{\pi}{3}}$$

سوال ② در بازه‌ی $[-\pi, \pi]$

$$\underbrace{5\sin^2(x)}_{1-\cos^2 x} + 2\cos(3x) = -2$$

سوال ③ در بازه‌ی $[0, 2\pi]$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$-\cos 2x = 2\sin^2 x - 1$$

$$2\sin(x)\cos(2x) + \sin x = 1$$

$$\sin x\,(\underbrace{2\cos 2x}_{(1-2\sin^2 x)} + 1) = 1$$

$$\sin x(2 - 4\sin^2 x + 1) = 1$$

$$(3 - 4\sin^2 x) = \frac{1}{\sin x}$$

سوال ④ در بازه‌ی $[0, \pi]$

$$\frac{1}{\sin\left(\frac{\pi}{2} + 2x\right)} + \frac{1}{\cos\left(\pi + \frac{\pi}{2} + 4x\right)} = 0$$

$$\frac{1}{\sin\left(\frac{\pi}{2} + 2x\right)} + \frac{1}{\cos\left(\frac{\pi}{2} + 4x\right)} = 0 \Rightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \Rightarrow \frac{1}{\cos 2x} = \frac{1}{2\sin 2x \cos 2x}$$

$$\cos 2x = 2\sin 2x \cos 2x \Rightarrow \sin 2x = \frac{1}{2} \Rightarrow \begin{cases} 2x = 2\pi + \frac{\pi}{6} \Rightarrow x = \pi + \frac{\pi}{12} \\ 2x = 2\pi + \frac{5\pi}{6} \Rightarrow x = \pi + \frac{5\pi}{12} \end{cases}$$

اختلاف $\Rightarrow \pi + \frac{5\pi}{12} - (\pi + \frac{\pi}{12}) = \frac{4\pi}{12} = \frac{\pi}{3} = \alpha$

$$\Rightarrow \tan 2\alpha = \tan\frac{2\pi}{3} = \boxed{-\sqrt{3}}$$

سوال ۸

در بازه $[0, 2\pi]$

$$4\sin\sin\left(\frac{3\pi}{4} - x\right) = 1 \Rightarrow -2\underbrace{\left(2\sin x\cos x\right)}_{\sin 2x} = 1$$

$$-\cos 3x$$

$$\sin 2x = -\frac{1}{2} \Rightarrow \boxed{2x = 2\pi - \frac{\pi}{6} \Rightarrow x = \pi - \frac{\pi}{12}}$$

$$\boxed{2x = 2\pi - \frac{5\pi}{6} \Rightarrow x = \pi - \frac{5\pi}{12}}$$

سوال ۹

در بازه $[0, \pi]$

$$\underbrace{\left(1+\cos(2\alpha)\right)}_{2\cos^2\alpha}\underbrace{\left(1+\cos(4\alpha)\right)}_{2\cos^2 2\alpha}\underbrace{\left(1+\cos(8\alpha)\right)}_{2\cos^2 4\alpha} = \frac{1}{8}$$

$$8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{8} \Rightarrow \cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{64}$$

$$\Rightarrow \cos\alpha\cos 2\alpha\cos 4\alpha = \frac{1}{8}$$

سوال ۱۰

در بازه $[0, 2\pi]$

$$\tan(3x) =$$

$$\tan(3x)\tan(x) = 1 \Rightarrow \underbrace{\tan(3x) = \cot(x)}$$

زاویای متمم، $\tan$ ، $\cot$ برابر دارند.

$$\Rightarrow 3x + x = 4x = \frac{\pi}{2} \Rightarrow \boxed{x = \frac{\pi}{8}}$$