نام و نام خانوادگی ...[illegible]... پاسخنامه تشریحی تکلیف شماره ..۱۷.. کلاس ...دوازدهم [illegible]...

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$8\cos x - \tan^2 x = 1 \Rightarrow 8\cos x = 1+\tan^2 x$

$1+\tan^2 x = 1+\frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x+\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \longrightarrow 8\cos x = \frac{1}{\cos^2 x}$

$\cos^3 x = \frac{1}{8}$

$\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3} \Rightarrow$ ۲ جواب

$\cos x \neq 0$ ✓

۲

$5\sin^2 x + 2\cos(3x) = -2 \xrightarrow[\cos 3x = 4\cos^3 x - 3\cos x]{\sin^2 x = 1-\cos^2 x} 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

چک کردن ۴ نقطه اصلی

$\rightarrow 0 \Rightarrow \cos 0 = 1 \Rightarrow 8-5-6+7 = 4$ ✗

$\rightarrow \frac{\pi}{2} \Rightarrow \cos\frac{\pi}{2} = 0 \Rightarrow 0-0-0+7 = 7$ ✗

$\rightarrow \pi \Rightarrow \cos\pi = -1 \Rightarrow -8-5+6+7 = 0$ ✓ $\Rightarrow$ به جز $2k\pi+\pi$ بقیه نقاط هستند! $\Rightarrow$ در $[-\pi, \pi]$ ۲ نقطه $\Rightarrow -\pi, \pi$

$\rightarrow \frac{3\pi}{2} \Rightarrow \cos\frac{3\pi}{2} = 0 \Rightarrow 7$ ✗

۳

$2\sin(x)\cos(2x) + \sin(x) = 1 \Rightarrow \sin x(2\cos 2x + 1) = 1 \xrightarrow{\cos 2x = 1-2\sin^2 x} \sin x(3-4\sin^2 x) = 1$

$3\sin x - 4\sin^3 x = 1 \Rightarrow \sin 3x = 1 \Rightarrow 3x = 2k\pi + \frac{\pi}{2} \Rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6}$

$K=-1 \rightarrow -\frac{2\pi}{3}+\frac{\pi}{6} = -\frac{\pi}{2}$ ✗

$K=0 \rightarrow \frac{\pi}{6}$

$K=1 \rightarrow \frac{2\pi}{3}+\frac{\pi}{6} = \frac{5\pi}{6}$

$K=2 \rightarrow \frac{4\pi}{3}+\frac{\pi}{6} = \frac{3\pi}{2}$

$K=3 \rightarrow 2\pi+\frac{\pi}{6} = \frac{13\pi}{6}$ ✗

$\Rightarrow \underbrace{\frac{\pi}{6}+\frac{5\pi}{6}}_{\pi} + \frac{3\pi}{2} = \frac{5\pi}{2}$

۴

$\frac{1}{\sin\left(\frac{\pi+4x}{2}\right)} + \frac{1}{\cos\left(\frac{\pi+8x}{2}\right)} = 0 \Rightarrow \frac{1}{\cos 2x} + \frac{1}{-2\sin 2x\cos 2x} = \frac{1}{\cos 2x}\left(\frac{2\sin 2x - 1}{2\sin 2x}\right) = 0$

$\sin\left(\frac{\pi+4x}{2}\right) = \sin\left(2x+\frac{\pi}{2}\right) = \cos 2x$

$\cos\left(\frac{\pi+8x}{2}\right) = \cos\left(4x+\frac{\pi}{2}\right) = -\sin 4x = -2\sin 2x\cos 2x$

$2\sin 2x = 1$

$\sin 2x = \frac{1}{2}$ $\rightarrow 2x = \frac{\pi}{6} \rightarrow x = \frac{\pi}{12}$; $2x = \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{12}$ $\xrightarrow{\text{اختلاف}} \frac{5\pi-\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3} = \alpha$

$\tan 2\alpha = \tan\frac{2\pi}{3} = \boxed{-\sqrt{3}}$

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$m(\cos x - \sin x) - 3\sqrt{6}\sin(2x) = \sqrt{6}$

$\boxed{\cos x - \sin x} = -(\sin x - \cos x) = -\sqrt{2}\sin\left(x-\frac{\pi}{4}\right) = \sqrt{2}\sin\left(\frac{\pi}{4}-x\right) = \sqrt{2}\cos\left(\frac{\pi}{4}+x\right) \Rightarrow \sqrt{2}\times\frac{1}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$

$\cos\left(x+\frac{\pi}{4}\right) = \frac{1}{\sqrt{3}} \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}} \Rightarrow \cos 2\alpha = 2\cos^2\alpha - 1 \Rightarrow \cos 2\alpha = 2\times\frac{1}{3} - 1 = -\frac{1}{3} \Rightarrow \cos\left(2x+\frac{\pi}{2}\right) = -\frac{1}{3}$

$\Rightarrow -\sin 2x = -\frac{1}{3} \Rightarrow \sin 2x = \frac{1}{3}$

عالیه

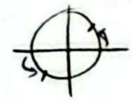
$\hookrightarrow m\times\frac{\sqrt{2}}{\sqrt{3}} - \cancel{3}\sqrt{6}\times\frac{1}{\cancel{3}} = \sqrt{6} \Rightarrow m = \frac{2\sqrt{6}\times\sqrt{3}}{\sqrt{2}} = \boxed{6}$

$\sin(x+\frac{\pi}{4})\cos(x-\frac{\pi}{4})=1 \Rightarrow \sin(x+\frac{\pi}{4})\cos(\frac{\pi}{4}-x)=1 \Rightarrow \cos^2(\frac{\pi}{4}-x)=1$

$x+\frac{\pi}{4}+\frac{\pi}{4}-x=\frac{\pi}{2}$

$\Rightarrow \cos(x-\frac{\pi}{4})=\pm1 \Rightarrow x-\frac{\pi}{4}=K\pi \Rightarrow x=K\pi+\frac{\pi}{4} \xrightarrow{[0,2\pi]}$ ۲ جواب دارد ۶

$(\frac{\pi}{4},\frac{5\pi}{4})$



$\sin x+\sqrt{3}\cos x=\sqrt{2} \Rightarrow \sin x\times\frac{1}{2}+\cos x\times\frac{\sqrt{3}}{2}=\frac{\sqrt{2}}{2} \Rightarrow \sin x\cos\frac{\pi}{3}+\cos x\sin\frac{\pi}{3}=\sin\frac{\pi}{4}$

$\Rightarrow \sin(x+\frac{\pi}{3})=\sin(\frac{\pi}{4})$

$\rightarrow x+\frac{\pi}{3}=2K\pi+\frac{\pi}{4} \Rightarrow x=2K\pi-\frac{\pi}{12} \xrightarrow{[-\pi,2\pi]} -\frac{\pi}{12}, \frac{23\pi}{12}$ ۷

$\rightarrow x+\frac{\pi}{3}=2K\pi+\underbrace{\pi-\frac{\pi}{4}}_{\frac{3\pi}{4}} \Rightarrow x=2K\pi+\frac{5\pi}{12} \xrightarrow{[-\pi,2\pi]} \frac{5\pi}{12}$

$\Rightarrow -\frac{\pi}{12}+\frac{23\pi}{12}+\frac{5\pi}{12}=\frac{27\pi}{12}=\boxed{\frac{9\pi}{4}}$

$4\sin x\underbrace{\sin(\frac{3\pi}{2}-x)}_{-\cos x}=1 \Rightarrow -4\sin x\cos x=1 \Rightarrow -2\sin 2x=1$

$\sin 2x=-\frac{1}{2}$

$\sin 2x=\sin(-\frac{\pi}{6})$

$\rightarrow 2x=2K\pi-\frac{\pi}{6} \Rightarrow x=K\pi-\frac{\pi}{12}$

$\rightarrow 2x=2K\pi+\frac{7\pi}{6} \Rightarrow x=K\pi+\frac{7\pi}{12}$

$x=K\pi-\frac{\pi}{12} \Rightarrow x=\frac{11\pi}{12},\frac{23\pi}{12}$

$x=K\pi+\frac{7\pi}{12} \Rightarrow x=\frac{7\pi}{12},\frac{19\pi}{12}$

$\rightarrow \frac{11\pi}{12}+\frac{23\pi}{12}+\frac{7\pi}{12}+\frac{19\pi}{12}=\boxed{5\pi}$

$\underbrace{(1+\cos 2\alpha)}_{2\cos^2\alpha}\underbrace{(1+\cos 4\alpha)}_{2\cos^2 2\alpha}\underbrace{(1+\cos 8\alpha)}_{2\cos^2 4\alpha}=\frac{1}{8} \Rightarrow \times\frac{\sin^2\alpha}{\sin^2\alpha}$ $\frac{8}{\sin^2\alpha}\Big(\underbrace{\underbrace{\sin^2\alpha\cos^2\alpha}_{\frac{1}{4}\sin^2 2\alpha}\cos^2 2\alpha}_{\frac{1}{16}\sin^2 4\alpha}\cos^2 4\alpha\Big)=\frac{1}{8}$

$\Rightarrow \frac{8}{\sin^2\alpha}\times\frac{1}{64}\sin^2 8\alpha=\frac{1}{8} \Rightarrow \sin^2 8\alpha=\sin^2\alpha \rightarrow \sin 8\alpha=\sin\alpha$

$\rightarrow 8\alpha=2K\pi+\alpha \Rightarrow \alpha=\frac{2K\pi}{7}$ ۹

$\rightarrow 8\alpha=2K\pi+\pi-\alpha \Rightarrow \alpha=\frac{2K\pi}{9}+\frac{\pi}{9}$

$\rightarrow \sin 8\alpha=\sin(-\alpha)$

$\rightarrow 8\alpha=2K\pi-\alpha \Rightarrow \alpha=\frac{2K\pi}{9}$

$\rightarrow 8\alpha=2K\pi+\pi+\alpha \Rightarrow \alpha=\frac{2K\pi}{7}+\frac{\pi}{7}$

$\alpha=\frac{2\pi}{7},\frac{4\pi}{7},\frac{6\pi}{7},\frac{\pi}{9},\frac{3\pi}{9},\frac{5\pi}{9},\frac{7\pi}{9},\frac{2\pi}{9},\frac{4\pi}{9},\frac{6\pi}{9},\underbrace{\boxed{\frac{8\pi}{9}}}_{max},\frac{\pi}{7},\frac{3\pi}{7},\frac{5\pi}{7}$

$\tan(3x)\tan(x)=1 \Rightarrow \tan x=\frac{1}{\tan 3x}=\cot 3x=\tan(\frac{\pi}{2}-3x)$

$\Rightarrow \tan x=\tan(\frac{\pi}{2}-3x)$

$x=K\pi+\frac{\pi}{2}-3x \Rightarrow x=\frac{K\pi}{4}+\frac{\pi}{8} \xrightarrow{[\pi,2\pi]} \frac{5\pi}{8}+\frac{7\pi}{8}=\frac{12\pi}{8}=\boxed{\frac{3\pi}{2}}$ ۱۰

(۱.۵۵)