

$$\lambda \cos \alpha = 1 + \tan \alpha = \frac{1}{\cos \alpha} \rightarrow \lambda \cos^2 \alpha = 1 \quad \text{فرم ۳}$$

$$\lambda \cos \alpha = 1 \rightarrow \cos \alpha = \frac{1}{\lambda} \rightarrow \alpha = 2k\pi \pm \arccos \frac{1}{\lambda} \quad \text{در بازه } [0, 2\pi] \quad \frac{\pi}{3} \text{ و } \frac{5\pi}{3}$$

۱ جواب

$$\lambda \sin^2 \alpha + \lambda \cos^2 \alpha = -\lambda \quad |\cos \alpha| \leq 1 \rightarrow -\lambda \leq \cos^2 \alpha \leq \lambda \quad \text{و } \sin^2 \alpha \geq 0$$

$$\rightarrow \sin \alpha = 0 \rightarrow \alpha = k\pi/5 \quad \lambda \cos^2 \alpha = -\lambda \rightarrow \cos^2 \alpha = -1 \rightarrow$$

$$\lambda \alpha = 2k\pi + \pi \rightarrow \alpha = \frac{2k\pi}{\lambda} + \frac{\pi}{\lambda} \quad \text{در بازه } [0, 2\pi] \quad \text{I و III}$$

۲ جواب

$$\lambda \sin \alpha \cos \lambda \alpha + \sin \alpha = 1 \quad | = \lambda \sin \alpha \cos \lambda \alpha \rightarrow \lambda \sin \alpha \cos \lambda \alpha + \sin \alpha - \lambda \sin \alpha - \cos \lambda \alpha = 0$$

$$\rightarrow \cos \lambda \alpha (\lambda \sin \alpha - 1) = \sin \alpha (\lambda \sin \alpha - 1) \quad \lambda \sin \alpha - 1 = 0 \rightarrow \cos \lambda \alpha = \sin \lambda \alpha = \cos(\frac{\pi}{2} - \lambda \alpha)$$

$$\lambda \sin \alpha - 1 = 0 \rightarrow \sin \alpha = \frac{1}{\lambda} \rightarrow \alpha = 2k\pi + \arcsin \frac{1}{\lambda} \quad \text{در بازه } [0, 2\pi] \quad \frac{\pi}{6} + \frac{\lambda\pi}{6} + \frac{\pi\pi}{6} = \frac{\lambda\pi}{6}$$

$$\lambda \alpha = 2k\pi + (\frac{\pi}{2} - \alpha) \rightarrow \alpha = \frac{2k\pi}{\lambda} + \frac{\pi}{2\lambda}$$

$$\lambda \alpha = 2k\pi - (\frac{\pi}{2} - \alpha) \rightarrow \alpha = \frac{2k\pi}{\lambda} - \frac{\pi}{2\lambda}$$

$$\frac{1}{\sin(\frac{\pi}{2} + \frac{\lambda\pi}{\lambda})} + \frac{1}{\cos(\frac{\pi}{2} + \frac{\lambda\pi}{\lambda})} = \frac{1}{\cos \lambda \alpha} + \frac{1}{\sin \lambda \alpha} = \frac{-\sin \lambda \alpha + \cos \lambda \alpha}{-\sin \lambda \alpha \cos \lambda \alpha} = 0$$

$$\frac{-\sin \lambda \alpha + \cos \lambda \alpha}{-\sin \lambda \alpha \cos \lambda \alpha} = 0 \rightarrow \sin \lambda \alpha = \cos \lambda \alpha \rightarrow \lambda \sin \alpha \cos \lambda \alpha = \cos \lambda \alpha \rightarrow \sin \lambda \alpha = 0$$

$$\sin \lambda \alpha = 0 \rightarrow \lambda \alpha = k\pi \rightarrow \alpha = \frac{k\pi}{\lambda}$$

$$\cos \lambda \alpha = 0 \rightarrow \lambda \alpha = \frac{\pi}{2} + k\pi \rightarrow \alpha = \frac{\pi}{2\lambda} + \frac{k\pi}{\lambda}$$

$$\rightarrow \alpha = \frac{2\pi}{19} - \frac{\pi}{19} = \frac{\pi}{19} \quad \tan \frac{\pi}{19} = -\sqrt{3}$$

$$\cos(\alpha + \frac{\pi}{2}) = \cos \alpha \cos \frac{\pi}{2} - \sin \alpha \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2} (\cos \alpha - \sin \alpha) = \frac{1}{\sqrt{2}}$$

$$\rightarrow \cos \alpha - \sin \alpha = \frac{2}{\sqrt{2}} \rightarrow \cos^2 \alpha + \sin^2 \alpha - \lambda \sin \alpha \cos \alpha = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}}$$

$$\rightarrow \sin \lambda \alpha = \frac{1}{\sqrt{2}} \quad \frac{\lambda m}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2} \rightarrow m = 6$$

۵

$$\sin(\alpha + \frac{\pi}{2}) \cos(\pi - \frac{\pi}{2}) = \sin(\alpha + \frac{\pi}{2}) \cos(\frac{\pi}{2} - \alpha) = 1 \quad \text{دوران مستقیم}$$

$$\sin^2(\alpha + \frac{\pi}{2}) = 1 \rightarrow \sin(\alpha + \frac{\pi}{2}) = \pm 1 \rightarrow \alpha + \frac{\pi}{2} = k\pi + \frac{\pi}{2} \rightarrow \alpha = k\pi + \frac{\pi}{2}$$

$$\text{حلول: } \frac{\pi}{2} \text{ و } \frac{5\pi}{2} \quad \text{بجای}$$

$$\sin \alpha + \tan \frac{\pi}{4} \cos \alpha = \frac{\sin \alpha \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos \alpha}{\cos \frac{\pi}{4}} = \frac{\sin(\alpha + \frac{\pi}{4})}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\rightarrow \sin(\alpha + \frac{\pi}{4}) = \frac{\sqrt{2}}{2} \Rightarrow \alpha = 2k\pi - \frac{3\pi}{4} \text{ یا } \alpha = 2k\pi + \frac{\pi}{4}$$

$$\text{حلول: } -\frac{\pi}{4} \text{ و } \frac{\pi}{4} \text{ و } \frac{7\pi}{4} \text{ و } \frac{5\pi}{4} \quad \text{یعنی} \quad \frac{5\pi}{4} = \frac{5\pi}{4}$$

$$- \sin \alpha \cos \alpha = 1 \rightarrow -2 \sin^2 \alpha = 1 \rightarrow \sin^2 \alpha = -\frac{1}{2}$$

$$2\alpha = 2k\pi + \frac{\pi}{2} \rightarrow \alpha = k\pi + \frac{\pi}{4}$$

$$2\alpha = 2k\pi + \frac{3\pi}{2} \rightarrow \alpha = k\pi + \frac{3\pi}{4}$$

$$\text{حلول: } \frac{\pi}{4} + \frac{3\pi}{4} = \frac{4\pi}{4} = \pi$$

$$1 + \cos 2\alpha = 2 \cos^2 \alpha \quad 1 + \cos 5\alpha = 2 \cos^2 \frac{5\alpha}{2} \quad 1 + \cos 11\alpha = 2 \cos^2 \frac{11\alpha}{2}$$

$$\frac{\sin^2 \alpha}{\sin^2 \frac{\alpha}{2}} \times 2 \cos^2 \frac{\alpha}{2} \times 2 \cos^2 \frac{\alpha}{2} \times 2 \cos^2 \frac{\alpha}{2} = \frac{\sin^2 \alpha}{\sin^2 \frac{\alpha}{2}} \times \cos^2 \frac{\alpha}{2} =$$

$$\frac{\sin^2 \alpha}{\sin^2 \frac{\alpha}{2}} = \frac{1}{\cos^2 \frac{\alpha}{2}} \rightarrow \sin^2 \alpha = \sin^2 \frac{\alpha}{2} \rightarrow \sin \alpha = \pm \sin \frac{\alpha}{2} \Rightarrow \alpha = \frac{2k\pi}{3} \quad \alpha = \frac{2k\pi}{9}$$

$$\text{حلول: } A: \left\{ \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9} \right\} \quad \text{و } \alpha = \frac{\pi}{9}$$

$$\tan 2\alpha = \frac{1}{\tan \alpha} = \cot \alpha = \tan(\frac{\pi}{2} - \alpha) \rightarrow 2\alpha = k\pi + \frac{\pi}{2} - \alpha \rightarrow$$

$$\alpha = \frac{k\pi}{3} + \frac{\pi}{3} \quad \text{حلول: } \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$\text{یعنی: } \frac{\pi + 2\pi + 4\pi + 5\pi}{3} = \frac{12\pi}{3} = 4\pi$$