هانیه ... نوزدهم دشتی A

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$8\cos x - \tan^2 x = 1 \to 1+\tan^2 x = \frac{1}{\cos^2}$

$8\cos x = \frac{1}{\cos^2} \to 8\cos^3 x = 1$

$\cos x = \frac{1}{8} \to \cos x = \frac{1}{2}$

$x = 2k\pi \pm \frac{\pi}{3} \to \frac{\pi}{3}, \frac{5\pi}{3}$

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$5\sin^2 x + 2\cos 3x = -2$

$5\sin^2 x + 2\cos 3x + 2 = 0$

$5\sin^2 x + 2(\cos 3x + 1) = 0$

$\cos 3x + 1 = 2\cos^2\frac{3x}{2}$

$\underbrace{5\sin^2 x} + 2\underbrace{(2\cos^2\frac{3x}{2})} = 0$

باید بین جواب ها اشتراک بگیری!

$5\sin x = 0 \to \sin x = 0 \to \boxed{x = k\pi}$ $\;0, \pi, -\pi$

$4\cos^2\frac{3x}{2} = 0 \to \cos\frac{3x}{2} = 0 = \frac{\pi}{2}$

$\pi, -\pi$

$\frac{3x}{2} = 2k\pi + \frac{\pi}{2} \to \boxed{\frac{4k\pi}{3} + \frac{\pi}{3}} \to \frac{\pi}{3}, -\pi$

$\frac{3x}{2} = 2k\pi - \frac{\pi}{2} \to \boxed{\frac{4k\pi}{3} - \frac{\pi}{3}} \to -\frac{\pi}{3}, \pi$

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$\cos 2x = 1 - 2\sin^2 x$

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$2\sin x\cos 2x + \sin x = 1$

$2\sin x(1 - 2\sin^2 x) + \sin x = 1$

$3\sin x - 4\sin^3 x = 1 \to \sin 3x = 1$

$3x = 2k\pi + \frac{\pi}{2} \to \frac{2k\pi}{3} + \frac{\pi}{6}$

$\boxed{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}} \to \frac{15\pi}{6}$

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$\frac{1}{\sin(\frac{\pi}{2}+3x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = 0$

$\cos(3x) \qquad -\sin(4x) = \sin(-4x)$

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$\sin(-4x) + \cos(3x) = 0 \to \cos(3x) = \sin(4x)$

$\cos(3x) = \cos(\frac{\pi}{2} - 4x)$

$\overset{9x}{\cancel{3x}} = 2k\pi + \frac{\pi}{2} - \cancel{4x} \to \frac{k\pi}{3} + \frac{\pi}{12} = \frac{4k\pi+\pi}{12}$

$\overset{-x}{\cancel{3x}} = 2k\pi - \frac{\pi}{2} + \cancel{4x} \to -k\pi + \frac{\pi}{4}$

$\frac{\pi}{4} \to \cos(\frac{\pi}{2}+\pi) = 0$ ✗

$\frac{3\pi}{4} \to \sin(\frac{\pi}{2}+\frac{3\pi}{2}) = 0$ ✗

$\cancel{\frac{\pi}{4}}$ $\quad \frac{\pi}{12}, \frac{5\pi}{12}, \cancel{\frac{9\pi}{12}}$

$\frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3} \qquad \tan(\frac{2\pi}{3}) = -\sqrt{3}$

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$m(\cos x - \sin x) - 2\sqrt{6}\sin(2x) = \sqrt{6}$

$-m(\sqrt{2}\sin(\cancel{x - \frac{\pi}{4}})) = \sqrt{6}(1 + 2\sin(2x))$

$\frac{1}{\sqrt{2}}$

$-m\sqrt{\frac{2}{18}} = 1 + 2\sin(2x)$

$\frac{-m}{3} = 1 + 2\sin(2x)$

$\to \sin(2x) = \frac{-3-m}{6}$

$\sin(x-\frac{\pi}{4}) = \cos(x+\frac{\pi}{4}) = \frac{1}{\sqrt{3}}$

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$\sin(x+\frac{\pi}{6})\cos(x-\frac{\pi}{3}) = 1$

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$\cos^2(x-\frac{\pi}{3}) = 1 \to \cos(x-\frac{\pi}{3}) = \pm 1$

$x - \frac{\pi}{3} = k\pi \to x = k\pi + \frac{\pi}{3} \qquad x = \frac{\pi}{3}, \frac{4\pi}{3}$

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$\sin x + \sqrt{3}\cos x = \sqrt{2} \qquad \alpha \to \tan\alpha = \frac{\sqrt{3}}{3}$

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$\frac{1}{1}\sqrt{1+3}\,\sin(x+\frac{\pi}{6})$

$2\sin(x+\frac{\pi}{6}) = \frac{\sqrt{2}}{2} = \sin\frac{\pi}{4}$

$x = 2k\pi + \frac{\pi}{4} - \frac{\pi}{6} = \frac{24k\pi + \pi}{12}$

$x = 2k\pi + \frac{3\pi}{4} - \frac{\pi}{6} = \frac{24k\pi + 7\pi}{12}$

$\frac{\pi}{12}$

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$-4\sin x\cos x = 1$

$2\sin x\cos x = \sin 2x = -\frac{1}{2}$

$2x = 2k\pi - \frac{\pi}{6} \to x = k\pi - \frac{\pi}{12}$

$2x = 2k\pi + \frac{5\pi}{6} \to x = k\pi + \frac{9\pi}{12}$

$x = \underbrace{\frac{11\pi}{12}, \frac{23\pi}{12}}_{2\pi + \frac{5\pi}{6}} : \underbrace{\frac{\pi}{2}, \frac{3\pi}{2}}_{+\ 2\pi} \quad 4\pi + \frac{5\pi}{6}$

هانیه هولی روز دهم دفتر A

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$$(1+\cos(2\alpha))(1+\cos(4\alpha))(1+\cos(8\alpha)) = \frac{1}{8}$$

$2\cos^2\alpha \qquad 2\cos^2 2\alpha \qquad 2\cos^2 4\alpha$

②

$$\to \cos^2\alpha \cdot \cos^2 2\alpha \cdot \cos^2 4\alpha = \frac{1}{64} \to (\underbrace{\cos\alpha \cos 2\alpha \cos 4\alpha}_{\frac{1}{8}})^2 = \frac{1}{64}$$

$$\sin\alpha\cos\alpha\cos 2\alpha\cos 4\alpha = \frac{1}{8}\sin\alpha$$

$\underbrace{\sin\alpha\cos\alpha}_{\frac{1}{2}\sin 2\alpha}$

$\frac{1}{4}\sin 4\alpha$

$\frac{1}{8}\sin 8\alpha \longrightarrow \sin\alpha = \sin 8\alpha$

$\alpha = 2k\pi + 8\alpha$

$-\cancel{7}\alpha = \frac{-2k\pi}{7} \xrightarrow{k=-2} \frac{4\pi}{7}$

$\alpha = 2k\pi - 8\alpha$

$9\alpha = \frac{2k\pi}{9} \xrightarrow{k=4} \frac{8\pi}{9}$ ✓

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$$\tan(3x)\tan(x) = 1 \to \tan(3x) = \frac{1}{\tan(x)} \to \tan(3x) = \cot(x) = \tan\left(\frac{\pi}{2} - x\right)$$

②

$$3x = k\pi + \frac{\pi}{2} - x \to 4x = k\pi - \frac{\pi}{2} \to x = \frac{k\pi}{4} - \frac{\pi}{8}$$

$\frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$ → 4 ✓

- ②

$\cos 3x = 2\cos^2\frac{3x}{2} - 1$ (فرمول طلایی)

$$5\sin^2 x + 2\left(2\cos^2\frac{3x}{2} - 1\right) = -2 \to 5\sin^2 x + 4\cos^2\frac{3x}{2} \cancel{-2} = \cancel{-2}$$

حاصل جمع دو عدد نامنفی صفر است پس هر دو صفر بوده اند!

$5\sin^2 x = 0 \to \sin x = 0 \to x = k\pi \xrightarrow{-\pi \le x \le \pi} x = \pi$ یا $-\pi$ یا 0

$4\cos^2\frac{3x}{2} = 0 \to \cos\frac{3x}{2} = 0 \to \frac{3x}{2} = k\pi + \frac{\pi}{2} \to x = \frac{2k\pi + \pi}{3}$

$\xrightarrow{x \le \pi} x = -\pi$ یا $-\frac{\pi}{3}$ یا $\frac{\pi}{3}$ یا π

اشتراک دو مجموعه جواب $\{\pi, -\pi\}$ است

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۵)

$$\cos(x+\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \to \cos x - \sin x = \frac{\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \to \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \to \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \to \frac{\sqrt{6}}{3}m - 3\sqrt{6}(\frac{1}{3}) = \sqrt{6}$$

$$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \to \frac{m\sqrt{6}}{3} = 2\sqrt{6} \to \boxed{m=6}$$

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۷) عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \to \sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \to x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0,1 \to x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \to x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0 \to x = \frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

$$\sin(\frac{3\pi}{2} - x) = -\cos x$$

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۸)

$$\sin(\frac{3\pi}{2} - x) = -\cos x$$

$$4\sin x(-\cos x) = 1 \to -2(2\sin x\cos x) = 1 \to \sin 2x = -\frac{1}{2}$$

$$2x = 2k\pi - \frac{\pi}{6} \text{ یا } 2k\pi + \frac{7\pi}{6} \to x = \overset{2}{k\pi - \frac{\pi}{12}} \text{ یا } \overset{1}{k\pi + \frac{7\pi}{12}}$$

$$\xrightarrow{0 \le x \le 2\pi} \begin{cases} 1:\ k=0,1 \to x = \frac{7\pi}{12}, \frac{19\pi}{12} \\ 2:\ k=1,2 \to x = \frac{11\pi}{12}, \frac{23\pi}{12} \end{cases}$$

$$S = \frac{7\pi}{12} + \frac{19\pi}{12} + \frac{11\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = \boxed{5\pi}$$