۱۳٫۷۵

آرینا حسینی — کلاس دوازدهم تجربی A — تکلیف شماره ۱۷

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$8\cos x - \tan^2 x = 1 \to 8\cos x = \underbrace{\tan^2 x + 1}_{\frac{1}{\cos^2 x}} \to 8\cos^3 x = 1 \to \cos^3 x = \frac{1}{8} \Rightarrow \cos x = \frac{1}{2}$ ۲

$\boxed{\frac{\pi}{3} \text{ و } \frac{5\pi}{3}}$

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$5\sin^2 x + 2\cos(3x) = -2 \to 5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) = -2$

$5 - 5\cos^2 x + 8\cos^3 x - 6\cos x = -2$

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$2\sin(x)\cos(2x) + \sin x = 1 \to \sin x(\underbrace{2\cos 2x}_{1-2\sin^2 x} + 1) = 1 \to \sin x\left(\frac{2-2\sin^2 x}{2\cos^2 x}\right) = 1$

$\underbrace{2\sin x\cos x}_{\sin 2x} \times \cos x = 1$

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۱٫۵

$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = \frac{1}{\cos 2x} - \frac{1}{\underbrace{\sin 4x}_{2\sin 2x\cos 2x}} = \frac{1\times 2\sin 2x - 1}{\cos 2x \times 2\sin 2x}$

$\frac{2\sin 2x - 1}{2\sin 2x\cos 2x} = 0 \to 2\sin 2x = 1 \to \sin 2x = \frac{1}{2} \to \sin 2x = \sin\frac{\pi}{6}$

$2x = 2k\pi + \frac{\pi}{6}$

$2x = 2k\pi + \pi - \frac{\pi}{6}$

$[0,\pi]$ در بازه $\to \frac{5\pi}{6} - \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3} \to \tan 2\times 120° = \tan 240° = \boxed{\sqrt{3}}$

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$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \to \frac{m\times\sqrt{6}}{3} - \cancel{3}\sqrt{6}\times\frac{1}{\cancel{3}} = \sqrt{6} \to m\sqrt{6} = 3 + 3\sqrt{6}$

$m = \frac{3+3\sqrt{6}\times\sqrt{6}}{\sqrt{6}\times\sqrt{6}} = \boxed{\frac{\sqrt{6}+6}{2}}$

$\cos(x+\frac{\pi}{4}) = \frac{1}{\sqrt{3}}$

$\cos x\cos\frac{\pi}{4} - \sin x\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}} \to \cos x - \sin x = \frac{2}{\sqrt{6}} \xrightarrow{\text{توان ۲}} \underbrace{\sin^2 x + \cos^2 x}_{1} - \sin 2x = \frac{4}{6}$

$\frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$

$\sin 2x = \frac{1}{3}$

**۶**

$$\sin(x+\overset{30}{\cancel{\tfrac{\pi}{6}}})\underbrace{\cos(x-\overset{60}{\cancel{\tfrac{\pi}{3}}})}_{\cos(\frac{\pi}{3}-x)}=1 \xrightarrow{\text{متمم هستند}} \sin(x+\tfrac{\pi}{6})^2=1$$

$\sin(x+\frac{\pi}{6})=1=\sin\frac{\pi}{2}$

$\sin(x+\frac{\pi}{6})=-1=\sin\frac{3\pi}{2}$

$x+\frac{\pi}{6}=2k\pi+\frac{\pi}{2} \rightarrow x=2k\pi+\frac{\pi}{3}$

$x+\frac{\pi}{6}=2k\pi+\cancel{\pi-\frac{\pi}{2}}\ \frac{\pi}{2}$

$x+\frac{\pi}{6}=2k\pi+\frac{3\pi}{2} \rightarrow x=2k\pi+\frac{4\pi}{3}$

$x+\frac{\pi}{6}=2k\pi+\underbrace{\cancel{\pi-\frac{3\pi}{2}}}_{-\frac{\pi}{2}} \rightarrow x=2k\pi-\frac{2\pi}{3}$

$\boxed{\frac{\pi}{3} \text{ و } \frac{4\pi}{3}}$ ← ۲ تا جواب دارد.

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**۷** ۱٫۵

$\sin x+\sqrt{3}\cos x=\sqrt{3} \rightarrow \sin x+\sqrt{3}\cos x=\frac{1}{|1|}\underbrace{\sqrt{1+3}}_{2}\sin(x+\frac{\pi}{3})=\sqrt{3}$

$\sin(x+\frac{\pi}{3})=\sin\frac{\pi}{4}$

$\tan\alpha=\sqrt{3} \rightarrow \tan\frac{\pi}{3}=\sqrt{3}$

$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \rightarrow x=2k\pi-\frac{\pi}{12}$ → ۲ جواب

$x+\frac{\pi}{3}=2k\pi+\underbrace{\cancel{\pi-\frac{\pi}{4}}}_{\frac{3\pi}{4}} \rightarrow x=2k\pi+\frac{5\pi}{12}$

مجموع؟

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**۸** ۱٫۵

$4\sin x-\cos x\, x=1 \rightarrow 4\sin x\cos x=-1 \rightarrow 2\sin 2x=-1 \rightarrow \sin 2x=-\frac{1}{2}$

$\sin 2x=\sin(-\frac{\pi}{6})$

$2x=2k\pi-\frac{\pi}{6}$ → ~~۲ تا جواب~~ ۴ جواب

$2x=2k\pi+\underbrace{\pi+\frac{\pi}{6}}_{\frac{7\pi}{6}}$

مجموع؟

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**۹** ۱٫۵

$\underbrace{(1+\cos 2\alpha)}_{2\cos^2\alpha}\underbrace{(1+\cos 4\alpha)}_{2\cos^2 2\alpha}\underbrace{(1+\cos 8\alpha)}_{2\cos^2 4\alpha}=\frac{1}{8} \longrightarrow 8(\cos\alpha\cos 2\alpha\cos 4\alpha)^2=\frac{1}{8}$

$\frac{\times\sin\alpha}{\times\sin\alpha}\ \cos\alpha\cos 2\alpha\cos 4\alpha=\frac{1}{8} \rightarrow \frac{(\frac{1}{2}\sin 2\alpha\cos 2\alpha)\cos 4\alpha}{\sin\alpha}=\frac{1}{8} \rightarrow \frac{\frac{1}{4}\sin 4\alpha\cos 4\alpha}{\sin\alpha}=\frac{1}{8}$

$\frac{\frac{1}{8}\sin 8\alpha}{\sin\alpha}=\frac{1}{8} \longrightarrow \sin 8\alpha=\sin\alpha$

$8\alpha=2k\pi+\alpha \rightarrow 7\alpha=2k\pi \rightarrow \alpha=\frac{2k\pi}{7}$

$8\alpha=2k\pi+\pi-\alpha \rightarrow \alpha=\frac{2}{9}k\pi+\frac{\pi}{9}$ → بررسی صفر مخرج $\pi$ ها

* A نمی‌تواند برابر خود π باشد چون طبق * sinα را مخالف صفر در نظر گرفته‌ایم!

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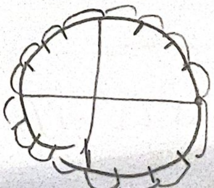
**۱۰** ۱٫۲۵

$\tan(3x)\tan(x)=1 \longrightarrow \tan 3x=\cot x \rightarrow \tan 3x=\tan(\frac{\pi}{2}-x)$

$\rightarrow 3x=k\pi+\frac{\pi}{2}-x \longrightarrow 4x=k\pi+\frac{\pi}{2} \rightarrow x=\frac{k\pi}{4}+\underbrace{\frac{\pi}{8}}_{22.5}$

مجموع جواب ها؟



→ ۸ تا جواب

بازه $[\pi, 2\pi]$

۲ -

$\cos 3x = 2\cos^2\frac{3x}{2} - 1$ (فرمول طلایی)

$5\sin^2 x + 2(2\cos^2\frac{3x}{2} - 1) = -2 \rightarrow 5\sin^2 x + 4\cos^2\frac{3x}{2} \cancel{-2} = \cancel{-2}$

حاصل جمع دو عدد نامنفی صفر است پس هر دو صفر بوده اند!

$5\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = k\pi \xrightarrow{-\pi \le x \le \pi} x = \pi$ یا $-\pi$ یا $0$

$4\cos^2\frac{3x}{2} = 0 \rightarrow \cos\frac{3x}{2} = 0 \rightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi + \pi}{3}$

$\xrightarrow{x \le \pi} x = -\pi$ یا $-\frac{\pi}{3}$ یا $\frac{\pi}{3}$ یا $\pi$

$\boxed{\text{اشتراک دو مجموعه جواب } \{-\pi, \pi\} \text{ است}}$

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$\cos 2x = 1 - 2\sin^2 x$ (فرمول طلایی)

$2\sin x(1 - 2\sin^2 x) + \sin x = 1 \rightarrow 3\sin x - 4\sin^3 x = 1$

۳ ٤ ۳

$\sin 3x = 1 \rightarrow 3x = 2k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \xrightarrow{0 \le x \le 2\pi}$

$k = 0 \rightarrow x = \frac{\pi}{6}$ , $k = 1 \rightarrow x = \frac{5\pi}{6}$ , $k = 2 \rightarrow x = \frac{9\pi}{6}$

$S = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$

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$\begin{cases} \sin(2x + \frac{\pi}{2}) = \cos 2x \\ \cos(\frac{\pi}{2} + 4x) = -\sin 4x \end{cases} \rightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \rightarrow \frac{\cos 2x - \sin 4x}{\cos 2x(-\sin 4x)} = 0$

$\cos 2x - 2\sin 2x\cos 2x = 0 \rightarrow \cos 2x(1 - 2\sin 2x) = 0$ $\rightarrow \cos 2x = 0$ ✗ ; $\rightarrow \sin 2x = \frac{1}{2}$ ✓

* اگر $\cos 2x = 0$ باشد مخرج عبارت اولیه صفر می‌شود که غیر قابل قبول است!

$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6}$ یا $2k\pi + \frac{5\pi}{6} \rightarrow x = k\pi + \frac{\pi}{12}$ یا $k\pi + \frac{5\pi}{12}$

$\xrightarrow{0 \le x \le \pi} k = 0 \rightarrow x = \frac{\pi}{12}$ یا $\frac{5\pi}{12} \rightarrow \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$

$\alpha = \frac{\pi}{3} \rightarrow \tan(2\alpha) = \tan(\frac{2\pi}{3}) = \boxed{-\sqrt{3}}$

۵)

$$\cos\left(x+\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \longrightarrow \cos x - \sin x = \frac{\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \longrightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \longrightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \longrightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}\left(\frac{1}{3}\right) = \sqrt{6}$$

$$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \longrightarrow \boxed{m=6}$$

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۷) عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi\le x\le 2\pi} k=0,1 \to x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \longrightarrow x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi\le x\le 2\pi} k=0 \to x = \frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

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۸)

$$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$$

$$4\sin x(-\cos x) = 1 \longrightarrow -2(2\sin x\cos x) = 1 \longrightarrow \sin 2x = -\frac{1}{2}$$

$$2x = 2k\pi - \frac{\pi}{6} \ \text{یا}\ 2k\pi + \frac{7\pi}{6} \longrightarrow x = \overset{2}{k\pi - \frac{\pi}{12}} \ \text{یا}\ \overset{1}{k\pi + \frac{7\pi}{12}}$$

$$\xrightarrow{0\le x\le 2\pi} \begin{cases} \overset{1}{} k=0,1 \longrightarrow x = \frac{7\pi}{12}, \frac{19\pi}{12} \\ \underset{2}{} k=1,2 \longrightarrow x = \frac{11\pi}{12}, \frac{23\pi}{12} \end{cases}$$

$$S = \frac{7\pi}{12} + \frac{19\pi}{12} + \frac{11\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = \boxed{5\pi}$$

(9)

$$1+\cos 2x = 2\cos^2 x$$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \xrightarrow{?} 8\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\,\cos 2\alpha\,\cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha \,\star}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \rightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \rightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \rightarrow x = \frac{5\pi}{7} \qquad \rightarrow \boxed{\max A = \frac{8\pi}{9}}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \rightarrow x = \frac{7\pi}{9}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ sinα را مخالف صفر در نظر گرفته‌ایم!

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(10)

$$\tan 3x = \frac{1}{\tan x} = \cot x \rightarrow \tan 3x = \tan\left(\frac{\pi}{2}-x\right)$$

$$3x = k\pi + \frac{\pi}{2} - x \rightarrow 4x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$$

| k | 4 | 5 | 6 | 7 |
|---|---|---|---|---|
| x | $\frac{9\pi}{8}$ | $\frac{11\pi}{8}$ | $\frac{13\pi}{8}$ | $\frac{15\pi}{8}$ |

$$S = \left(\frac{9+11+13+15}{8}\right)\pi = \frac{48\pi}{8} = \boxed{6\pi}$$