حسابان - ۱۲ دفتر A

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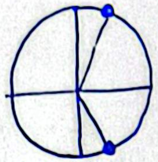
①

$8\cos\alpha - \tan^2\alpha = 1 \qquad 8\cos\alpha = 1+\tan^2\alpha \qquad 8\cos\alpha = \frac{1}{\cos^2\alpha} \qquad \cos^3\alpha = \frac{1}{8}$

$\cos\alpha = \frac{1}{2}$



۲ جواب

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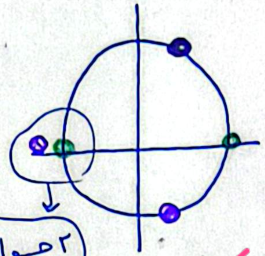
②

$5\sin^2\alpha + 2\cos(3\alpha) = -2$

$$\begin{cases} \sin^2\alpha = 0 \longrightarrow \alpha = k\pi \\ \cos 3\alpha = -1 \longrightarrow 3\alpha = 2k\pi + \pi \qquad \alpha = \frac{2k\pi}{3} + \frac{\pi}{3} \end{cases}$$

جواب های کلی $= \frac{\pi}{3}$ و π , $\frac{5\pi}{3}$

روی دایره



۲ جواب دارد

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③

$2\sin(\alpha)\,\cancel{\cos(2\alpha)} + \sin(\alpha) = 1$

$1-2\sin^2\alpha$

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$2\sin\alpha - 4\sin^3\alpha + \sin\alpha = 1 \qquad 3\sin\alpha - 4\sin^3\alpha = 1 \qquad -\sin 3\alpha = 1$

$\sin 3\alpha = -1 \qquad 3\alpha = 2k\pi + \frac{3\pi}{2} \qquad \alpha = \frac{2k\pi}{3} + \frac{\pi}{2}$

k	0	1	2
α	$\frac{\pi}{2}$	$\frac{7\pi}{6}$	$\frac{11\pi}{6}$

$\frac{\pi + 7\pi + 11\pi}{6} = \boxed{\frac{19\pi}{6}}$

یا $\frac{4k\pi}{6} + \frac{3\pi}{6}$

(4)

$$\frac{1}{\sin\left(\frac{\pi+4x}{2}\right)}+\frac{1}{\cos\left(\frac{\pi+8x}{2}\right)}=0 \qquad \frac{1}{\cos 2x}=\frac{1}{\sin 4x} \qquad \sin 4x=\cos 2x$$

$$2\sin 2x\cos 2x-\cos 2x=0 \qquad \cos 2x(2\sin 2x-1)=0 \to \cos 2x=0$$ غ ق ق
دو مخرج را صفر می‌کند

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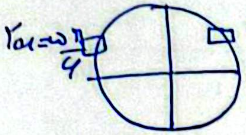
$$\to 2\sin 2x=1$$

$$\sin 2x=\frac{1}{2}$$



$$\alpha=x_2-x_1$$

$$\frac{5\pi}{12}-\frac{\pi}{12}\to\frac{\pi}{3} \qquad \tan\left(\frac{2\pi}{3}\right)=-\sqrt{3}$$

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(6)

$$\sin\left(x+\frac{\pi}{6}\right)\cos\underbrace{\left(x-\frac{\pi}{3}\right)}_{\frac{\pi}{3}-x}=1 \qquad \cancel{x}+\frac{\pi}{6}+\frac{2\pi}{6}-\cancel{x}\to\frac{\pi}{2}\to\sin x=\cos x$$

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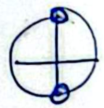
$$\sin\left(x+\frac{\pi}{6}\right)\sin\left(x+\frac{\pi}{6}\right)=1 \qquad \sin^2\left(x+\frac{\pi}{6}\right)=1 \qquad \sin\left(x+\frac{\pi}{6}\right)=\pm 1$$



$$x+\frac{\pi}{6}=k\pi+\frac{\pi}{2} \qquad x=k\pi+\frac{\pi}{3}\xrightarrow{\text{جواب‌ها}}$$

k	0	1
x	$\frac{\pi}{3}$	$\frac{4\pi}{3}$

۲ جواب

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(7)

$$\sin x+\sqrt{3}\cos x=\sqrt{2}\to\frac{a}{|a|}\sqrt{a^2+b^2}\sin(x+\alpha)\to 2\sin\left(x+\frac{\pi}{3}\right)=\sqrt{2}$$

$$\tan\alpha=\frac{b}{a} \qquad \tan\alpha=\sqrt{3} \qquad \alpha=60^\circ$$

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$$\sin\left(x+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2} \qquad x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \qquad x=2k\pi-\frac{\pi}{12}$$

k	0	1
x	$-\frac{\pi}{12}$	$\frac{23\pi}{12}$

$$x+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4} \qquad x=2k\pi+\frac{5\pi}{12}$$

k	0	1
x	$\frac{5\pi}{12}$	$\cancel{\frac{29\pi}{12}}$

$$\left\}\ \frac{23\pi}{12}-\frac{\pi}{12}+\frac{5\pi}{12}\right.$$

$$\frac{27\pi}{12}$$

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$$4\sin\alpha \sin\Big(\underbrace{\tfrac{3\pi}{2}-\alpha}_{-\cos\alpha}\Big)=1 \qquad -4\,\underbrace{\sin\alpha\cos\alpha}_{\frac{1}{2}\sin 2\alpha}=1 \qquad -2\sin 2\alpha=1$$

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$$\sin 2\alpha=-\frac{1}{2} \qquad 2\alpha=2k\pi-\frac{\pi}{6} \qquad \alpha=k\pi-\frac{\pi}{12}$$

k	1	2
α	$\frac{11\pi}{12}$	$\frac{23\pi}{12}$

$$2\alpha=2k\pi+\frac{7\pi}{6} \qquad \alpha=k\pi+\frac{7\pi}{12}$$

k	0	1
α	$\frac{7\pi}{12}$	$\frac{19\pi}{12}$

$$\frac{11\pi+23\pi+7\pi+19\pi}{12}=\frac{60\pi}{12}=5\pi$$

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$$\underbrace{(1+\cos 2\alpha)}_{2\cos^2\alpha}\cdot\underbrace{(1+\cos(4\alpha))}_{2\cos^2 2\alpha}\cdot\underbrace{(1+\cos(8\alpha))}_{2\cos^2 4\alpha}=\frac{1}{8}$$

$$(\cos 2\alpha\times\cos\alpha\times\cos 4\alpha)^2=\frac{1}{64}$$

۱,۵

$$\cos\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha=\frac{1}{8}\ \xrightarrow[\times\sin\alpha]{\times\sin\alpha}$$

$$\frac{\overbrace{\overbrace{\overbrace{\sin\alpha\cos\alpha}^{\frac{1}{2}\sin 2\alpha}\cdot\cos 2\alpha}^{\frac{1}{4}\sin 4\alpha}\cdot\cos 4\alpha}^{\frac{1}{8}\sin 8\alpha}}{\sin\alpha}$$

$$\to\ \frac{1}{8}\,\frac{\sin 8\alpha}{\sin\alpha}=\frac{1}{8} \qquad \sin 8\alpha=\sin\alpha \qquad 8\alpha=2k\pi+\alpha \qquad \alpha=\frac{2k\pi}{7}$$

$$8\alpha=2k\pi+\pi-\alpha \qquad \alpha=\frac{2k\pi}{9}+\frac{\pi}{9}$$

$$\alpha=\frac{2k\pi}{7}$$

k	0	1	2	3
α	0	$\frac{2\pi}{7}$	$\frac{4\pi}{7}$	$\frac{6\pi}{7}$

$$\frac{2k\pi}{9}+\frac{\pi}{9}$$

k	0	1	2	3	4
α	$\frac{\pi}{9}$	$\frac{3\pi}{9}$	$\frac{5\pi}{9}$	$\frac{7\pi}{9}$	$\frac{9\pi}{9}$

$\frac{9\pi}{9}$ ← بزرگ‌ترین عضو

* A نمی‌تواند برابر خود π باشد چون طبق * sinα را مخالف صفر در نظر گرفته‌ایم!

$\tan(3x)\,\cancel{\tan(x)}=1 \qquad \tan(x)=\frac{1}{\cot x}$ $\qquad \tan(3x)=\cot(x) \qquad \tan(3x)=\tan\left(\frac{\pi}{2}-x\right)$ ۱۰

$3x = k\pi + \frac{\pi}{2} - x \qquad x=\frac{k\pi}{4}+\frac{\pi}{8}$

k	۴	۵	۶	۷	~~۸~~
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$	$\cancel{\frac{17\pi}{8}}$

$\frac{9\pi+11\pi+13\pi+15\pi}{8} \qquad \frac{\cancel{38}\,48\,\pi}{8} \rightarrow \cancel{\frac{19\pi}{4}} \qquad 6\pi$ ارفاق

۵

$$\cos(x+\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \longrightarrow \cos x - \sin x = \frac{\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \longrightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \longrightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \longrightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}(\frac{1}{3}) = \sqrt{6}$$

$$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \longrightarrow \boxed{m=6}$$

۳

$$\cos 2x = 1 - 2\sin^2 x$$ (فرمول طلایی)

۳ ۴ ۳

$$2\sin x(1-2\sin^2 x) + \sin x = 1 \longrightarrow 3\sin x - 4\sin^3 x = 1$$

$$\sin 3x = 1 \longrightarrow 3x = 2k\pi + \frac{\pi}{2} \longrightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \xrightarrow{0 \leq x \leq 2\pi}$$

$$k=0 \longrightarrow x = \frac{\pi}{6}, \quad k=1 \longrightarrow x = \frac{5\pi}{6}, \quad k=2 \longrightarrow x = \frac{9\pi}{6}$$

$$S = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$$

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$$1+\cos 2\star = 2\cos^2\star$$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \xrightarrow{?} 8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\cos 2\alpha\cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha \ \star}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\cos 4\alpha = \pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \longrightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا} \ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا} \ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \longrightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \longrightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \longrightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \longrightarrow x = \frac{7\pi}{9}$$

$$\longrightarrow \boxed{\max A = \frac{8\pi}{9}}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ sinα را مخالف صفر در نظر گرفته‌ایم!