مریم ترکمانی (دوازدهم A صبحه تکلیف ۳

سوال ۱)

$8\cos x - \tan^2 x = 1 \quad [0, 2\pi]$

$8\cos x = \underbrace{\tan^2 x + 1}$

$\hookrightarrow \frac{\sin^2 x}{\cos^2 x} + 1 = \frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$\to 8\cos x = \frac{1}{\cos^2 x} \to \cos^3 x = \frac{1}{8} \to \cos x = \frac{1}{2}$

$\cos x = \frac{1}{2} \to x \in 2k\pi \pm \frac{\pi}{3} \nearrow x = \frac{\pi}{3}, \frac{5\pi}{3} \Rightarrow$ $\boxed{\text{۲ جواب}}$

سوال ۲)

$5\sin^2 x + 2\cos 3x = -2 \quad [-\pi, \pi]$

$\sin^2 x = 1 - \cos^2 x \ , \ \cos 3x = 4\cos^3 x - 3\cos x$

$\rightsquigarrow 8\cos^3 x - 6\cos x - 5\cos^2 x + 7 = 0$

بررسی ۴ نقطه اصلی:

$x = 0 \to 8 - 6 - 5 + 7 \neq 0$ ×

$x = \frac{\pi}{2} \to 0 + 0 + 0 + 7 \neq 0$ × $\Rightarrow x = \frac{3\pi}{2}$ نیز همین جواب را می‌دهد

$x = \pi \to -8 + 6 - 5 + 7 = 0$ ✓ $\to x \in 2k\pi \pm \pi$

$\to$ جواب‌ها $= \pi, -\pi \Rightarrow$ $\boxed{2}$

سوال ۳)

$2\sin x\cos 2x+\sin x=1 \qquad [0,2\pi]$

$\to \sin x(2\cos 2x+1)=1 \xrightarrow{\cos 2x=1-2\sin^2 x} \sin x(3-4\sin^2 x)=1$

$\to 3\sin x-4\sin^3 x=1 \Rightarrow \sin 3x=1 \to 3x\in 2k\pi+\frac{\pi}{2}$

$\to x\in \frac{2k\pi}{3}+\frac{\pi}{6} \to \boxed{\frac{\pi}{6}},\ \boxed{\frac{5\pi}{6}},\ \boxed{\frac{3\pi}{2}} \to$ مجموع $=\boxed{\frac{5\pi}{2}}$

سوال ۴)

$\frac{1}{\sin\left(\frac{\pi+4x}{2}\right)}+\frac{1}{\cos\left(\frac{\pi+8x}{2}\right)}=0 \qquad \tan 2\alpha=?$

$\to \sin\left(\frac{\pi}{2}+2x\right)=-\cos\left(\frac{\pi}{2}+4x\right) \Rightarrow \cos 2x=\sin 4x$

$\to \cancel{\cos 2x}=2\sin 2x\cancel{\cos 2x} \xrightarrow{\cos 2x\neq 0} 2\sin 2x=1 \to \sin 2x=\frac{1}{2}$

$2x\in 2k\pi+\frac{\pi}{6} \quad , \quad 2k\pi+\frac{5\pi}{6}$

$x\in k\pi+\frac{\pi}{12} \to \frac{\pi}{12} \qquad x\in k\pi+\frac{5\pi}{12} \to \frac{5\pi}{12} \xrightarrow{\text{اختلاف}} \frac{4\pi}{12}=\frac{\pi}{3}$

$\alpha=\frac{\pi}{3} \to 2\alpha=\frac{2\pi}{3}$

$\tan\frac{2\pi}{3}=\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\boxed{-\sqrt{3}}$

سوال ۵)

$m\underbrace{(\cos x-\sin x)}-2\sqrt{6}\sin 2x=\sqrt{6} \qquad \cos\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}} \qquad m=?$

$-(\sin x-\cos x)=-\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=\sqrt{2}\sin\left(\frac{\pi}{4}-x\right)$

$=\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{\sqrt{3}}$

* ادامه حل در صفحه بعد

$\cos(x+\frac{\pi}{4}) = \frac{1}{\sqrt{3}}$ ~ $\cos\alpha = \frac{1}{\sqrt{3}} \Rightarrow \cos 2\alpha = 2\cos^2\alpha - 1$

$\Rightarrow \cos 2\alpha = \frac{2}{3} - 1 = -\frac{1}{3} \longrightarrow \cos(2x+\frac{\pi}{2}) = -\frac{1}{3} \Rightarrow -\sin 2x = -\frac{1}{3}$

$\Rightarrow \sin 2x = \frac{1}{3}$

جایگذاری $\rightarrow m(\frac{\sqrt{2}}{\sqrt{3}}) - \cancel{2}\sqrt{6} \times \frac{1}{\cancel{3}} = \sqrt{6} \Rightarrow \frac{m\sqrt{\cancel{2}}}{\sqrt{3}} = \frac{2\sqrt{\cancel{6}}}{2\sqrt{3}}$

$\Rightarrow \boxed{m=6}$

سوال ۶) $\sin(x+\frac{\pi}{6})\cos(x-\frac{\pi}{3}) = 1 \quad [0, 2\pi]$

$\underbrace{\cos(x-\frac{\pi}{3})}_{\cos(\frac{\pi}{3}-x)}$

دو زاویه متمم $\rightarrow \sin(x+\frac{\pi}{6}) = \cos(\frac{\pi}{3}-x)$

$\rightarrow \sin^2(x+\frac{\pi}{6}) = 1 \longrightarrow \sin(x+\frac{\pi}{6}) = \pm 1$

$\rightarrow x+\frac{\pi}{6} = k\pi \pm \frac{\pi}{2} \longrightarrow x = k\pi + \frac{\pi}{3} \rightarrow \frac{\pi}{3}, \frac{4\pi}{3}$

$x = k\pi - \frac{2\pi}{3} \rightarrow \frac{\pi}{3}, \frac{4\pi}{3}$ } جواب

سوال ۷) $\sin x + \sqrt{3}\cos x = \sqrt{2} \quad [-\pi, 2\pi]$

$\rightarrow \frac{1}{2} \times \sin x + \frac{\sqrt{3}}{2} \times \cos x = \frac{\sqrt{2}}{2} \Rightarrow \sin(x+\frac{\pi}{3}) = \sin(\frac{\pi}{4})$

$x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \rightarrow x = 2k\pi - \frac{\pi}{12} \rightarrow -\frac{\pi}{12}, \frac{23\pi}{12}$

$x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \rightarrow x = 2k\pi + \frac{5\pi}{12} \rightarrow \frac{5\pi}{12}$

~ $\boxed{\text{مجموع} = \frac{9\pi}{4}}$

سوال ۸)

$$\underbrace{۴\sin x \sin\left(\frac{۳\pi}{۲}-x\right)}_{-\cos x}=۱ \qquad [۰,۳\pi]$$

$$\rightarrow -۴\sin x\cos x=۱ \Rightarrow -۲\sin ۲x=۱ \rightarrow \sin ۲x=-\frac{۱}{۲}$$

$$\rightarrow ۲x = ۲k\pi-\frac{\pi}{۶} \rightarrow x = k\pi-\frac{\pi}{۱۲} \rightarrow \frac{۱۱\pi}{۱۲}, \frac{۲۳\pi}{۱۲}$$

$$\searrow ۲x = ۲k\pi+\frac{۷\pi}{۶} \rightarrow x=k\pi+\frac{۷\pi}{۱۲}\rightarrow \frac{۷\pi}{۱۲},\frac{۱۹\pi}{۱۲}$$

$$\rightsquigarrow \boxed{\text{مجموع}=۵\pi}$$

سوال ۹)

$$(۱+\cos ۲\alpha)(۱+\cos ۴\alpha)(۱+\cos ۸\alpha)=\frac{۱}{۸} \qquad [۰,\pi]$$

$$۱+\cos ۲\alpha = \cancel{\sin^۲\alpha}+\cos^۲\alpha+\cos^۲\alpha-\cancel{\sin^۲\alpha}$$

$$=۲\cos^۲\alpha$$

$$\rightarrow (۲\cos^۲\alpha)(۲\cos^۲ ۲\alpha)(۲\cos^۲ ۴\alpha)=\frac{۱}{۸}$$

$$\rightarrow \cos^۲\alpha\cos^۲ ۲\alpha\cos^۲ ۴\alpha=\frac{۱}{۶۴}\rightarrow \cos\alpha\cos ۲\alpha\cos ۴\alpha=\pm\frac{۱}{۸}$$

$$\times\frac{\sin\alpha}{\sin\alpha} \rightarrow \frac{\frac{۱}{۸}\sin ۸\alpha}{\sin\alpha}=\pm\frac{۱}{۸}\rightarrow \frac{\sin ۸\alpha}{\sin\alpha}=\pm۱$$

$$\rightarrow \sin ۸\alpha=\sin\alpha \rightarrow ۸\alpha=۲k\pi+\alpha\rightarrow\alpha=\frac{۲k\pi}{۷}$$

$$\searrow ۸\alpha=۲k\pi+\pi-\alpha \rightarrow \alpha=\frac{۲k\pi}{۹}+\frac{\pi}{۹}$$

$$\searrow \sin ۸\alpha=\sin(-\alpha)$$

$$\searrow ۸\alpha=۲k\pi-\alpha\rightarrow\alpha=\frac{۲k\pi}{۹}$$

$$\searrow ۸\alpha=۲k\pi+\alpha+\pi\rightarrow\alpha=\frac{۲k\pi}{۷}+\frac{\pi}{۷}$$

$$\rightsquigarrow \max_{\text{جواب}}=\boxed{\frac{۸\pi}{۹}}$$

Subject:
Year. Month. Date.

سوال ۱۰

$\tan 3x \tan x = 1 \qquad [\pi , 2\pi]$

$\tan 3x = \frac{1}{\tan x} \longrightarrow \cot 3x = \tan x \longrightarrow$ x و 3x زاویه‌ی متمم

$\cot 3x = \tan(\frac{\pi}{2} - 3x)$

$\longrightarrow \tan x = \tan(\frac{\pi}{2} - 3x) \longrightarrow x = k\pi + \frac{\pi}{2} - 3x \rightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$

$x = \frac{9\pi}{8} , \frac{11\pi}{8} , \frac{13\pi}{8} , \frac{15\pi}{8}$

$\longrightarrow$ مجموع $= 6\pi$