SUBJECT: عربری

Year: Month: Day: Page: ( )

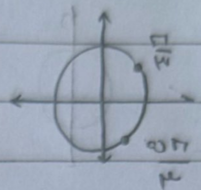
۱۵/۱۵

1) $8\cos\alpha - \tan^2 x = 1 \qquad [0, 2\pi]$

۲

$8\cos\alpha = 1 + \tan^2\alpha \rightarrow 8\cos\alpha = \frac{1}{\cos^2\alpha} \rightarrow \cos^3\alpha = \frac{1}{8}$

$\rightarrow \cos\alpha = \frac{1}{2} \rightarrow \begin{cases} \alpha = 2k\pi + \frac{\pi}{3} \\ \alpha = 2k\pi - \frac{\pi}{3} \end{cases}$



$\rightarrow$ جوابها: $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ ۲ جواب دارد

2) $5\sin^2(x) + 2\cos(3x) = -2 \qquad [-\pi, \pi]$

.۱۵

$5 - 5\cos^2\alpha$

$-5\cos^2\alpha + 2\cos(3\alpha) + 7 = 0$

$\cos 3x = 2\cos^2\frac{3x}{2} - 1$ (فرمول طلایی)

$5\sin^2 x + 2(2\cos^2\frac{3x}{2} - 1) = -2 \rightarrow 5\sin^2 x + 4\cos^2\frac{3x}{2} \cancel{-2} = \cancel{-2}$

حاصل جمع دو عدد نامنفی صفر است پس هر دو صفر بوده اند!

$5\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = k\pi \xrightarrow{-\pi \le x \le \pi} x = \pi$ یا $-\pi$ یا 0

$4\cos^2\frac{3x}{2} = 0 \rightarrow \cos\frac{3x}{2} = 0 \rightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi + \pi}{3}$

$\xrightarrow{x \le \pi} x = -\pi$ یا $-\frac{\pi}{3}$ یا $\frac{\pi}{3}$ یا π

$\boxed{\text{اشتراک مجموعه جواب } \{\pi, -\pi\} \text{ است}}$

3) $2\sin\alpha \overbrace{\cos 2\alpha}^{(1 - 2\sin^2\alpha)} + \sin\alpha = 1$

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$2\sin\alpha - 4\sin^3\alpha + \sin\alpha = 1 \rightarrow 3\sin\alpha \underbrace{- 4\sin^3\alpha}_{-3\sin^3\alpha - \sin^3\alpha} - 1 = 0$

$3\sin\alpha(1 - \sin^2\alpha) = (\cancel{\sin\alpha + 1})(\sin^2\alpha + 1 - \sin\alpha)$

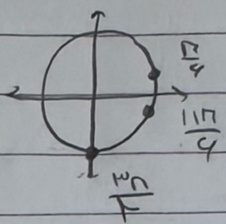
$(\cancel{1 + \sin})(1 - \sin) \quad \rightsquigarrow \quad \sin\alpha + 1 = 0$

$\sin\alpha = -1$

$3\sin\alpha - 3\sin^2\alpha = \sin^2\alpha + 1 - \sin\alpha$

$4\sin^2\alpha - 4\sin\alpha + 1 = 0$

$(2\sin\alpha - 1)^2 = 0 \rightarrow \sin\alpha = \frac{1}{2}$



$$\frac{\pi}{6}+\frac{11\pi}{6}+\frac{9\pi}{6}=\frac{21\pi}{6}$$

$$\frac{1}{\sin\left(\frac{\pi}{2}+4x\right)}+\frac{1}{\cos\left(\frac{\pi}{2}+8x\right)}=0 \quad ۱۴$$

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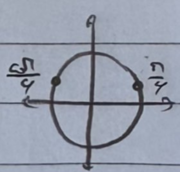
$$\frac{1}{\sin 2x}+\frac{1}{\underbrace{-\sin(4x)}_{-2\sin 2x\cos 2x}}=0$$

$$\frac{1}{\cancel{\sin 2x}}=\frac{1}{2\cancel{\sin 2x}\cos 2x} \rightarrow \sin x\neq 0 \rightarrow x\neq k\pi$$

$$\cos 2x=\frac{1}{2} \rightarrow \begin{cases} 2x=2k\pi+\frac{\pi}{3} \rightarrow x=k\pi+\frac{\pi}{6} \\ 2x=2k\pi-\frac{\pi}{3} \rightarrow x=k\pi-\frac{\pi}{6} \end{cases}$$



$$\frac{5\pi}{12}-\frac{\pi}{12}=\frac{\pi}{3} \longrightarrow \tan\frac{2\pi}{3}=-\sqrt{3}$$

$$\frac{5\pi}{6}-\frac{\pi}{6}=\frac{4\pi}{6}=\frac{2\pi}{3}=\alpha$$

$$\tan\frac{4\pi}{12}=\sqrt{3}$$

۵۱) $m(\cos\alpha - \sin\alpha) - ۳\sqrt{۶}\sin ۲\alpha = \sqrt{۶}$

۲

$\cos(x + \frac{\pi}{۴}) = \frac{۱}{\sqrt{۳}} \rightarrow \frac{\sqrt{۲}}{۲}(\cos\alpha - \sin\alpha) = \frac{۱}{\sqrt{۳}}$

$m = ?$ $\qquad \cos\alpha - \sin\alpha = \frac{۲}{\sqrt{۶}} = \frac{\sqrt{۶}}{۳}$

$\frac{\sqrt{۶}}{۳} m - ۳\sqrt{۶}\sin ۲\alpha = \sqrt{۶} \rightarrow \frac{m}{۳} - ۳\sin ۲\alpha = ۱$

(از بالاتر محاسبه شد) $\cos\alpha - \sin\alpha = \frac{\sqrt{۶}}{۳} \xrightarrow{\text{توان } ۲} ۱ - \sin ۲\alpha = \frac{۶}{۹} \rightarrow \sin ۲\alpha = \frac{۳}{۹} = \frac{۱}{۳}$

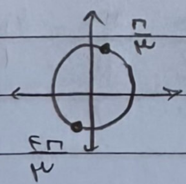
$\frac{m}{۳} - ۱ = ۱ \rightarrow m = ۶$

۶۱) $\underbrace{\sin(x + \frac{\pi}{۶})}_{\cos(\frac{\pi}{۳} - x)} \cos(x - \frac{\pi}{۳}) = ۱ \qquad [۰, ۲\pi]$

۲

داخلشان جمعشان $\frac{\pi}{۲}$ هست $\rightarrow \cos^۲(\frac{\pi}{۳} - x) = ۱ \rightarrow \cos^۲(x - \frac{\pi}{۳}) = ۱$

$\rightarrow \cos(x - \frac{\pi}{۳}) = \pm ۱ \rightarrow x - \frac{\pi}{۳} = k\pi \rightarrow x = k\pi + \frac{\pi}{۳}$



ارفاق

جواب‌ها $= \{ -\frac{\pi}{۳} , \frac{\pi}{۳} , \frac{۴\pi}{۳} \}$ ✗ ✓ ✓

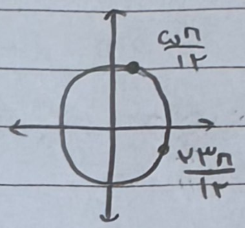
$-\frac{\pi}{۳}$ در بازه‌ی سوال نیست!

7) $\sin x + \sqrt{3}\cos x = \sqrt{2} \qquad [-\pi , 2\pi]$

1,5

$$\xrightarrow{\div 2} \quad \frac{\sin x}{2} + \frac{\sqrt{3}\cos x}{2} = \frac{\sqrt{2}}{2} \longrightarrow \cos\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x - \frac{\pi}{6} = 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi + \frac{\pi}{4} + \frac{\pi}{6} \\ x - \frac{\pi}{6} = 2k\pi - \frac{\pi}{4} \longrightarrow x = 2k\pi - \frac{\pi}{4} + \frac{\pi}{6} \end{cases}$$



جوابها $= \left\{ \cancel{-\frac{5\pi}{12}}\ -\frac{\pi}{12} , \frac{5\pi}{12} , \frac{23\pi}{12} \right\}$

مجموع جوابها : $\cancel{\frac{23\pi}{12}} \quad \frac{27\pi}{12}$ یا $\frac{9\pi}{4}$

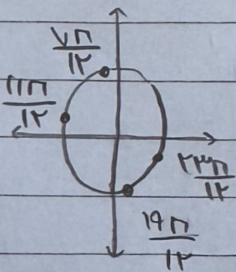
8) $4\sin x \sin\left(\underbrace{\frac{3\pi}{2} - x}_{-\cos x}\right) = 1 \qquad [0 , 2\pi]$

2

$-4\sin x \cos x = 1$

$\sin 2x = \frac{-1}{2}$

$$\begin{cases} 2x = 2k\pi - \frac{\pi}{6} \longrightarrow x = k\pi - \frac{\pi}{12} \\ 2x = 2k\pi - \frac{5\pi}{6} \longrightarrow x = k\pi - \frac{5\pi}{12} \end{cases}$$



جمع : $\frac{7\pi}{12} + \frac{11\pi}{12} + \frac{19\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = 5$

۹۱ $(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\frac{1}{8} \qquad [0,\pi]$

۱٫۵

$$\underbrace{1+\cos 2\alpha}_{2\cos^2\alpha},\quad \underbrace{1+\cos 4\alpha}_{2\cos^2 2\alpha},\quad \underbrace{1+\cos 8\alpha}_{2\cos^2 4\alpha}$$

$\times\sin^2\alpha \longrightarrow$ $\div \sin^2\alpha$

$$\frac{\sin^2\alpha\times 2\cos^2\alpha\times 2\cos^2 2\alpha\times 2\cos^2 4\alpha}{\sin^2\alpha}=\frac{2\sin^2 2\alpha\cos^2 2\alpha\cos\ldots}{\sin^2\alpha}$$

$\times 2^3 \longrightarrow$ $\div 2^3$

$$\frac{2\times 2\times\boxed{2\times 2\sin^2 2\alpha\cos^2 2\alpha}\cos^2 4\alpha}{8\times\sin^2\alpha}=\frac{4\sin^2 4\alpha\cos^2 4\alpha}{8\times\sin^2\alpha}$$

$$\frac{\sin^2 8\alpha}{8\sin^2\alpha}=\frac{1}{8}\longrightarrow \sin^2 8\alpha=\sin^2\alpha \longrightarrow \sin 8\alpha=\pm\sin\alpha$$

$$\longrightarrow\begin{cases}\sin 8\alpha=\sin\alpha\\ \sin 8\alpha=\sin(-\alpha)\end{cases}$$

$$\begin{cases}8\alpha=2k\pi+\alpha\\ \alpha=\frac{2k\pi}{7}\\ 8\alpha=2k\pi+\pi-\alpha\\ \alpha=\frac{2k\pi}{9}+\frac{\pi}{9}\end{cases}\quad \text{یا} \quad \begin{cases}8\alpha=2k\pi-\alpha\\ \alpha=\frac{2k\pi}{9}\\ 8\alpha=2k\pi+\pi+\alpha\\ \alpha=\frac{2k\pi}{7}+\frac{\pi}{7}\end{cases}$$

$\alpha=\frac{2k\pi}{7}$: $k=0\to\alpha=0$, $k=1\to\alpha=\frac{2\pi}{7}$, $k=2\to\alpha=\frac{4\pi}{7}$, $k=3\to\alpha=\frac{6\pi}{7}$

$\alpha=\frac{2k\pi}{9}$: $k=0\to\alpha=0$, $k=1\to\alpha=\frac{2\pi}{9}$, $k=2\to\alpha=\frac{4\pi}{9}$, $k=3\to\alpha=\frac{2\pi}{3}$, $k=4\to\alpha=\frac{8\pi}{9}$

$\alpha=\frac{2k\pi}{9}+\frac{\pi}{9}$: $k=0\to\alpha=\frac{\pi}{9}$, $k=1\to\alpha=\frac{\pi}{3}$, $k=2\to\alpha=\frac{5\pi}{9}$, $k=3\to\alpha=\frac{7\pi}{9}$, $k=4\to\alpha=\pi$

$\alpha=\frac{2k\pi}{7}+\frac{\pi}{7}$: $\alpha=0\to\frac{\pi}{7}$, $\alpha=1\to\frac{3\pi}{7}$, $\alpha=2\to\frac{5\pi}{7}$, $\alpha=3\to\pi$

★ A نمی‌تواند برابر خود π یا ۰ باشد چون طبق ★ sinα را مخالف صفر در نظر گرفته‌ایم!

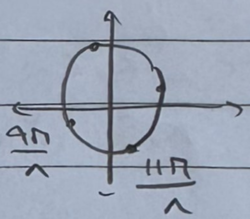
Max عدد $\longrightarrow \pi$

۱۰) $\tan ۳\alpha \tan \alpha = ۱ \quad [\pi , ۲\pi]$

۱,۲۵

$\tan ۳\alpha = \frac{۱}{\tan \alpha} = \cot \alpha \rightarrow \tan ۳\alpha = \tan(\frac{\pi}{۲} - \alpha)$

$۳\alpha = k\pi + \frac{\pi}{۲} - \alpha \rightarrow \alpha = \frac{k\pi}{۴} + \frac{\pi}{۸}$



→ جواب‌ها → در دامنه هستند

$\Sigma : \frac{۹\pi}{۸} + \frac{۱۱\pi}{۸} = \frac{۲۰\pi}{۸} = \frac{۵\pi}{۲}$

$\tan ۳x = \frac{۱}{\tan x} = \cot x \rightarrow \tan ۳x = \tan(\frac{\pi}{۲} - x)$ ۱۰

$۳x = k\pi + \frac{\pi}{۲} - x \rightarrow ۴x = k\pi + \frac{\pi}{۲} \rightarrow x = \frac{k\pi}{۴} + \frac{\pi}{۸}$

k	۴	۵	۶	۷
x	$\frac{۹\pi}{۸}$	$\frac{۱۱\pi}{۸}$	$\frac{۱۳\pi}{۸}$	$\frac{۱۵\pi}{۸}$

$S = (\frac{۹+۱۱+۱۳+۱۵}{۸})\pi = \frac{۴۸\pi}{۸} = \boxed{۶\pi}$

۳

$\cos 2x = 1 - 2\sin^2 x$ (فرمول طلایی)

$2\sin x(1-2\sin^2 x) + \sin x = 1 \longrightarrow 3\sin x - 4\sin^3 x = 1$

$\sin 3x = 1 \longrightarrow 3x = 2k\pi + \frac{\pi}{2} \longrightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \xrightarrow{0 \le x \le 2\pi}$

$k=0 \to x = \frac{\pi}{6}$, $k=1 \to x = \frac{5\pi}{6}$, $k=2 \to x = \frac{9\pi}{6}$

$S = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$

۴

$$\begin{cases} \sin(2x + \frac{\pi}{2}) = \cos 2x \\ \cos(\frac{\pi}{2} + 4x) = -\sin 4x \end{cases} \longrightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \to \frac{\cos 2x - \sin 4x}{\cos 2x(-\sin 4x)} = 0$$

$\cos 2x - 2\sin 2x \cos 2x = 0 \longrightarrow \cos 2x(1 - 2\sin 2x) = 0$

$\to \cos 2x = 0$ ✗

$\to \sin 2x = \frac{1}{2}$ ✓

* اگر $\cos 2x = 0$ باشد مخرج عبارت اولیه صفر می‌شود که غیر قابل قبول است!

$\sin 2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6}$ یا $2k\pi + \frac{5\pi}{6} \longrightarrow x = k\pi + \frac{\pi}{12}$ یا $k\pi + \frac{5\pi}{12}$

$\xrightarrow{0 \le x \le \pi} k=0 \longrightarrow x = \frac{\pi}{12}$ یا $\frac{5\pi}{12} \longrightarrow \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$

$\alpha = \frac{\pi}{3} \longrightarrow \tan(2\alpha) = \tan(\frac{2\pi}{3}) = \boxed{-\sqrt{3}}$

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۷ عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$

$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin(x + \frac{\pi}{3}) = \frac{\sqrt{2}}{2}$

$$\begin{cases} x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0,1 \to x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \longrightarrow x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0 \to x = \frac{5\pi}{12} \end{cases}$$

$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$

$1 + \cos 2x = 2\cos^2 x$ ★

$$\left.\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases}\right\} \rightarrow 8\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha \cos 2\alpha \cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha} \bigstar$$

$$\underbrace{(\sin\alpha \cos\alpha)}_{\sin 2\alpha} \cos 2\alpha \cos 4\alpha = \pm \frac{\sin\alpha}{8} \rightarrow \frac{1}{2} \underbrace{(\sin 2\alpha \cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha} \cos 4\alpha = \pm \frac{\sin\alpha}{8}$$

$$\frac{1}{2} \times \frac{1}{2} \underbrace{(\sin 4\alpha \cos 4\alpha)}_{\sin 8\alpha} = \frac{\pm \sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}} \sin 8\alpha = \pm \frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm \sin\alpha \longrightarrow \begin{cases} 8\alpha = 2k\pi + \alpha \ \text{یا} \ 2k\pi + \pi - \alpha \\ 8\alpha = 2k\pi - \alpha \ \text{یا} \ 2k\pi + \pi + \alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k = 3 \longrightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k = 4 \longrightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k = 2 \longrightarrow x = \frac{5\pi}{7} \qquad \longrightarrow \boxed{\max A = \frac{8\pi}{9}}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k = 3 \longrightarrow x = \frac{7\pi}{9}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ sinα را مخالف صفر در نظر گرفته‌ایم!