SUBJECT: عربری

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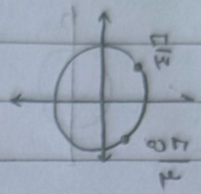
1) $8\cos\alpha - \tan^2 x = 1$ $[0, 2\pi]$

$8\cos\alpha = 1 + \tan^2\alpha \rightarrow 8\cos\alpha = \frac{1}{\cos^2\alpha} \rightarrow \cos^3\alpha = \frac{1}{8}$

$\rightarrow \cos\alpha = \frac{1}{2} \rightarrow \begin{cases} \alpha = 2k\pi + \frac{\pi}{3} \\ \alpha = 2k\pi - \frac{\pi}{3} \end{cases}$



$\rightarrow$ جوابها : $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ ۲ جواب دارد

2) $\underbrace{5\sin^2(x)}_{5 - 5\cos^2\alpha} + 2\cos(3x) = -2$ $[-\pi, \pi]$

$-5\cos^2\alpha + 2\cos(3\alpha) + 7 = 0$

3) $2\sin\alpha \overbrace{\cos 2\alpha}^{(1 - 2\sin^2\alpha)} + \sin\alpha = 1$

$2\sin\alpha - 4\sin^3\alpha + \sin\alpha = 1 \longrightarrow 3\sin\alpha \underbrace{- 4\sin^3\alpha}_{-3\sin^3\alpha - \sin^3\alpha} - 1 = 0$

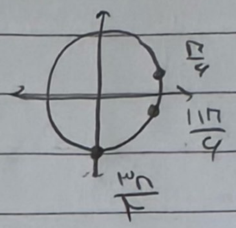
$3\sin\alpha\underbrace{(1 - \sin^2\alpha)}_{(\cancel{1+\sin})(1 - \sin)} = (\cancel{\sin\alpha + 1})(\sin^2\alpha + 1 - \sin\alpha)$ $\rightsquigarrow$ $\sin\alpha + 1 = 0$

$\sin\alpha = -1$

$3\sin\alpha - 3\sin^2\alpha = \sin^2\alpha + 1 - \sin\alpha$

$4\sin^2\alpha - 4\sin\alpha + 1 = 0$

$(2\sin\alpha - 1)^2 = 0 \rightarrow \sin\alpha = \frac{1}{2}$



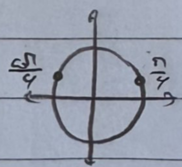
$$\frac{\pi}{6} + \frac{11\pi}{6} + \frac{9\pi}{6} = \frac{21\pi}{6}$$

$$\frac{1}{\sin\left(\frac{\pi + 4x}{2}\right)} + \frac{1}{\cos\left(\frac{\pi + 8x}{2}\right)} = 0 \quad 14$$

$$\frac{1}{\sin 2x} + \frac{1}{\underbrace{-\sin(4x)}_{-2\sin 2x \cos 2x}} = 0$$

$$\frac{1}{\cancel{\sin 2x}} = \frac{1}{2\cancel{\sin 2x}\cos 2x} \to \sin x \neq 0 \to x \neq k\pi$$

$$\cos 2x = \frac{1}{2} \to \begin{cases} 2x = 2k\pi + \frac{\pi}{3} \to x = k\pi + \frac{\pi}{6} \\ 2x = 2k\pi - \frac{\pi}{3} \to x = k\pi - \frac{\pi}{6} \end{cases}$$



$$\frac{5\pi}{6} - \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3} = a$$

$$\tan \frac{2\pi}{3} = \sqrt{3}$$

الف) $m(\cos\alpha - \sin\alpha) - 3\sqrt{6}\sin 2\alpha = \sqrt{6}$

$\cos(x + \frac{\pi}{4}) = \frac{1}{\sqrt{3}} \rightarrow \frac{\sqrt{2}}{2}(\cos\alpha - \sin\alpha) = \frac{1}{\sqrt{3}}$

$m = ?$ $\qquad \cos\alpha - \sin\alpha = \frac{2}{\sqrt{6}} = \frac{\sqrt{6}}{3}$

$\frac{\sqrt{6}}{3}m - 3\sqrt{6}\sin 2\alpha = \sqrt{6} \rightarrow \frac{m}{3} - 3\sin 2\alpha = 1$

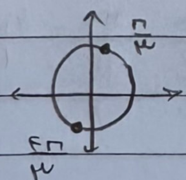
(با اتحاد محاسبه شد) $\cos\alpha - \sin\alpha = \frac{\sqrt{6}}{3} \xrightarrow{\text{توان } 2} 1 - \sin 2\alpha = \frac{6}{9} \rightarrow \sin 2\alpha = \frac{3}{9} = \frac{1}{3}$

$\frac{m}{3} - 1 = 1 \rightarrow m = 6$

ب) $\sin(x + \frac{\pi}{6})\underbrace{\cos(x - \frac{\pi}{3})}_{\cos(\frac{\pi}{3} - x)} = 1 \qquad [0, 2\pi]$

دو زاویه متمم هستند $\frac{\pi}{2}$ $\rightarrow \cos^2(\frac{\pi}{3} - x) = 1 \rightarrow \cos^2(x - \frac{\pi}{3}) = 1$

$\rightarrow \cos(x - \frac{\pi}{3}) = \pm 1 \rightarrow x - \frac{\pi}{3} = k\pi \rightarrow x = k\pi + \frac{\pi}{3}$

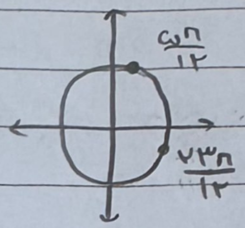


جواب‌ها $= \{-\frac{\pi}{3}, \frac{\pi}{3}, \frac{4\pi}{3}\}$

۷۱) $\sin x + \sqrt{۳}\cos x = \sqrt{۲}$ $[-\pi , ۲\pi]$

$\xrightarrow{\div \frac{۱}{۲}}$ $\frac{\sin x}{۲} + \frac{\sqrt{۳}\cos x}{۲} = \frac{\sqrt{۲}}{۲} \rightarrow \cos\left(x - \frac{\pi}{۶}\right) = \frac{\sqrt{۲}}{۲}$

$$\begin{cases} x - \frac{\pi}{۶} = ۲k\pi + \frac{\pi}{۴} \rightarrow x = ۲k\pi + \frac{\pi}{۴} + \frac{\pi}{۶} \\ x - \frac{\pi}{۶} = ۲k\pi - \frac{\pi}{۴} \rightarrow x = ۲k\pi - \frac{\pi}{۴} + \frac{\pi}{۶} \end{cases}$$



جوابها $= \left\{ -\frac{۵\pi}{۱۲} , \frac{۵\pi}{۱۲} , \frac{۲۳\pi}{۱۲} \right\}$

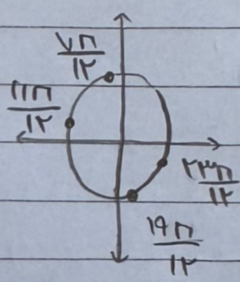
مجموع جوابها : $\frac{۲۳\pi}{۱۲}$

۸۱) $۴\sin x \sin\left(\underbrace{\frac{۳\pi}{۲} - x}_{-\cos x}\right) = ۱$ $[۰, ۲\pi]$

$-۴\sin x \cos x = ۱$

$\sin ۲x = \frac{-۱}{۲}$

$$\begin{cases} ۲x = ۲k\pi - \frac{\pi}{۶} \rightarrow x = k\pi - \frac{\pi}{۱۲} \\ ۲x = ۲k\pi - \frac{۵\pi}{۶} \rightarrow x = k\pi - \frac{۵\pi}{۱۲} \end{cases}$$



مجـ : $\frac{۷\pi}{۱۲} + \frac{۱۱\pi}{۱۲} + \frac{۱۹\pi}{۱۲} + \frac{۲۳\pi}{۱۲} = \frac{۶۰\pi}{۱۲} = ۵$

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۹۱ $(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\frac{1}{8} \qquad [0,\pi]$

$\underbrace{1+\cos2\alpha}_{2\cos^2\alpha}\quad \underbrace{1+\cos4\alpha}_{2\cos^2 2\alpha}\quad \underbrace{1+\cos8\alpha}_{2\cos^2 4\alpha}$

$\xrightarrow[\div \sin^2\alpha]{\times \sin^2\alpha} \dfrac{\sin^2\alpha\times 2\cos^2\alpha\times 2\cos^2 2\alpha\times 2\cos^2 4\alpha}{\sin^2\alpha}=\dfrac{2\sin^2 2\alpha\cos^2 2\alpha\cos\dots}{\sin^2\alpha}$

$\xrightarrow[\div 2^3]{\times 2^3} \dfrac{2\times2\times\boxed{2\times2\sin^2 2\alpha\cos^2 2\alpha}\cos^2 4\alpha}{8\times\sin^2\alpha}=\dfrac{4\sin^2 4\alpha\cos^2 4\alpha}{8\times\sin^2\alpha}$

$\dfrac{\sin^2 8\alpha}{8\sin^2\alpha}=\dfrac{1}{8}\longrightarrow \sin^2 8\alpha=\sin^2\alpha \rightarrow \sin 8\alpha=\pm\sin\alpha$

$\rightarrow\begin{cases}\sin 8\alpha=\sin\alpha\\ \sin 8\alpha=\sin(-\alpha)\end{cases}$

$\begin{cases}8\alpha=2k\pi+\alpha\\ \alpha=\frac{2k\pi}{7}\\ 8\alpha=2k\pi+\pi-\alpha\\ \alpha=\frac{2k\pi}{9}+\frac{\pi}{9}\end{cases}$ ل $\begin{cases}8\alpha=2k\pi-\alpha\\ \alpha=\frac{2k\pi}{9}\\ 8\alpha=2k\pi+\pi+\alpha\\ \alpha=\frac{2k\pi}{7}+\frac{\pi}{7}\end{cases}$

$\alpha=\frac{2k\pi}{7}$: $k=0\rightarrow\alpha=0$, $k=1\rightarrow\alpha=\frac{2\pi}{7}$, $k=2\rightarrow\alpha=\frac{4\pi}{7}$, $k=3\rightarrow\alpha=\frac{6\pi}{7}$

$\alpha=\frac{2k\pi}{9}$: $k=0\rightarrow\alpha=0$, $k=1\rightarrow\alpha=\frac{2\pi}{9}$, $k=2\rightarrow\alpha=\frac{4\pi}{9}$, $k=3\rightarrow\alpha=\frac{2\pi}{3}$, $k=4\rightarrow\alpha=\frac{8\pi}{9}$

$\alpha=\frac{2k\pi}{9}+\frac{\pi}{9}$: $k=0\rightarrow\alpha=\frac{\pi}{9}$, $k=1\rightarrow\alpha=\frac{\pi}{3}$, $k=2\rightarrow\alpha=\frac{5\pi}{9}$, $k=3\rightarrow\alpha=\frac{7\pi}{9}$, $k=4\rightarrow\alpha=\pi$

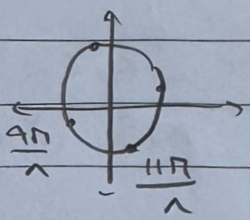
$\alpha=\frac{2k\pi}{7}+\frac{\pi}{7}$: $\alpha=0\rightarrow\frac{\pi}{7}$, $\alpha=1\rightarrow\frac{3\pi}{7}$, $\alpha=2\rightarrow\frac{5\pi}{7}$, $\alpha=3\rightarrow\frac{7}{\pi}$

Max des $\rightarrow \pi$

۱۵) $\tan 3\alpha \tan \alpha = 1 \quad [\pi, 2\pi]$

$$\tan 3\alpha = \frac{1}{\tan \alpha} = \cot \alpha \rightarrow \tan 3\alpha = \tan\left(\frac{\pi}{2} - \alpha\right)$$

$$\left\{ 3\alpha = k\pi + \frac{\pi}{2} - \alpha \rightarrow \alpha = \frac{k\pi}{4} + \frac{\pi}{8} \right.$$



→ جواب‌های خاص → در دامنه هستند

$$\Sigma : \frac{9\pi}{8} + \frac{11\pi}{8} = \frac{20\pi}{8} = \frac{5\pi}{2}$$