نام و نام خانوادگی ......خانم اسماعیل‌زاده...... پاسخنامه تشریحی تکلیف شماره ...۱۷... کلاس دوازدهم ریاضی A جبرها

$8\cos x = \tan^2 x + 1 \to 8\cos x = \frac{1}{\cos^2 x} \to \cos^3 x = \frac{1}{8} \to \cos x = \frac{1}{2} \to$

$x = 2k\pi \pm \frac{\pi}{3}$ این معادله در بازه $[0, 2\pi]$ ۲ جواب $\frac{\pi}{3}$ ، $\frac{5\pi}{3}$ دارد. ۱

$5\sin^2 x + 2\cos 3x + 2 = 0 \xrightarrow[\cos 3x = 4\cos^3 x - 3\cos x]{\sin^2 x = 1 - \cos^2 x} 5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) + 2 = 0 \to$

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0 \xrightarrow{\text{جمع ضرایب توان فرد = زوج}} \cos x = -1 \to$ ۲

$(\cos x + 1)\underbrace{(8\cos^2 x - 13\cos x + 7)}_{\Delta<0 \text{ ریشه ندارد}} = 0 \to \cos x = -1 \to x = 2k\pi - \pi$

در بازه $[-\pi, \pi]$ دو جواب $\{-\pi, \pi\}$ را دارد. (۲ جواب)

$\sin x(2\cos 2x + 1) = 1 \xrightarrow{\cos 2x = 1 - 2\sin^2 x} \sin x(2 - 4\sin^2 x + 1) = 1 \to$

$4\sin^3 x - 3\sin x + 1 = 0 \xrightarrow{\text{جمع ضرایب فرد = زوج}} \sin x = -1 : (\sin x + 1)(4\sin^2 x - 4\sin x + 1) \to$

$(\sin x + 1)(2\sin x - 1)^2 = 0 \to \sin x = \{-1, \frac{1}{2}\} \to x = \{2k\pi - \frac{\pi}{2}, 2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}\}$ ۳

در بازه $[0, 2\pi]$ جواب‌های $\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\}$ قابل قبول است. که مجموع آن‌ها $\frac{5\pi}{2}$ می‌باشد.

$\frac{1}{\sin(2x+\frac{\pi}{2})} + \frac{1}{\cos(4x+\frac{\pi}{2})} = 0 \to \frac{1}{\cos 2x} + \frac{-1}{\sin 4x} = 0 \to \frac{1}{\cos 2x} = \frac{1}{\sin 4x} \to$

$\cos 2x = \sin 4x \to \cos 2x = 2\sin 2x \cos 2x \to \sin 2x = \frac{1}{2} \to 2x = \{2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}\} \to$ ۴

$x = \{k\pi + \frac{\pi}{12}, k\pi + \frac{5\pi}{12}\} \to$ در بازه $[0, \pi]$ $\to x = \frac{\pi}{12}, \frac{5\pi}{12} \to \alpha = \frac{5\pi - \pi}{12} = \frac{\pi}{3} \to$

$\tan\frac{2\pi}{3} = -\sqrt{3}$

$\cos(x + \frac{\pi}{4}) = \frac{1}{\sqrt{3}} \to \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}} \to \cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}}$

اتحادی: $(\cos x - \sin x)^2 = 1 - \sin 2x \to \sin 2x = 1 - \frac{2}{3} = \frac{1}{3}$

جایگذاری: $\frac{\sqrt{2}}{\sqrt{3}}(m) - 3\sqrt{6}(\frac{1}{3}) = \sqrt{6} \to \frac{\sqrt{2}}{\sqrt{3}} m = 2\sqrt{6} \to \boxed{m = 6}$ ۵

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$\sin(x+\frac{\pi}{4})\cos(x-\frac{\pi}{3}) = 1 \rightarrow 2\cos(x-\frac{\pi}{3}) = 1 \rightarrow \cos(x-\frac{\pi}{3}) = \frac{1}{2} \rightarrow$

$x-\frac{\pi}{3} = 2k\pi \pm \frac{\pi}{3} \rightarrow x = \{2k\pi + \frac{2\pi}{3} , 2k\pi\}$

تعداد جواب معادله در بازه‌ی $[0 , 2\pi]$ ۳ تا خواهد بود.

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$\sin x + \frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}\cos x = \sqrt{2} \rightarrow \frac{\cos\frac{\pi}{3}\sin x + \sin\frac{\pi}{3}\cos x}{\underline{\cos\frac{\pi}{3}}\ \frac{1}{2}} = \sqrt{2} \rightarrow$

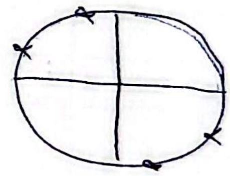
$\sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2} \rightarrow x+\frac{\pi}{3} = \{2k\pi+\frac{\pi}{4} , 2k\pi+\frac{3\pi}{4}\} \rightarrow$

$x = \{2k\pi - \frac{\pi}{12} , 2k\pi + \frac{5\pi}{12}\} \xrightarrow{\text{در بازه } (-\pi, 2\pi]} x = \{-\frac{\pi}{12} , \frac{23\pi}{12} , \frac{5\pi}{12}\} \xrightarrow{\text{جمع}} \frac{9\pi}{4}$

۸

$4\sin x \cdot \underbrace{\sin(\frac{3\pi}{2}-x)}_{-\cos x} = 1 \rightarrow -2\sin 2x = 1 \rightarrow \sin 2x = \frac{-1}{2} \rightarrow$

$2x = \{2k\pi - \frac{\pi}{6} , 2k\pi - \frac{5\pi}{6}\} \rightarrow x = \{k\pi - \frac{\pi}{12} , k\pi - \frac{5\pi}{12}\}$



۴ جواب در بازه‌ی $[0 , 2\pi]$ داریم.

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$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha) = \frac{1}{8} \rightarrow (2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha) = \frac{1}{8}$

از طرفی داریم: $\sin 8\alpha = 8\sin\alpha\cos\alpha\cos 2\alpha\cos 4\alpha$ $\rightarrow$

$\frac{|\sin 8\alpha|}{|\sin\alpha|} = 1 \rightarrow \sin 8\alpha = \sin\alpha$ یا $\sin 8\alpha = \sin(-\alpha) \Rightarrow \alpha = \{\frac{2k\pi}{9} , \frac{2k\pi}{7} , \frac{2k\pi+\pi}{7} , \frac{2k\pi+\pi}{9}\} \rightarrow$

در بازه $[0, \pi]$: $\alpha = \{\frac{2\pi}{7}, \frac{4\pi}{7}, \frac{6\pi}{7}, \frac{\pi}{7}, \frac{3\pi}{7}, \frac{5\pi}{7}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{6\pi}{9}, \frac{8\pi}{9}, \frac{\pi}{9}, \frac{3\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}\}$

$\boxed{\frac{8\pi}{9} = \alpha}$

۱۰

$\tan 3x \cdot \tan x = 1 \rightarrow \tan 3x = \cot x \rightarrow \tan 3x = \tan(\frac{\pi}{2} - x) \rightarrow$

$3x = k\pi + \frac{\pi}{2} - x \rightarrow 4x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{4} + \frac{\pi}{8} \longrightarrow$ در بازه: $[\pi, 2\pi]$

$x = \{\frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\} \longrightarrow \boxed{\text{جمع} , 6\pi}$