۱۹٫۵ آزین بوستا

نام و نام خانوادگی ..دستانیس..۵...معمارزاده... پاسخنامه تشریحی تکلیف شماره ...۱۷.. کلاس ..دوازدهم ریاضی..A....

**۱**

$8\cos x - \tan^2 x = 1 \to 8\cos x = 1 + \tan^2 x \to 8\cos x = \frac{1}{\cos^2 x}$

$1 + \tan^2 x = 1 + \frac{\sin^2}{\cos^2} = \frac{1}{\cos^2 x}$

$\cos^3 x = \frac{1}{8}$

$\cos x = \frac{1}{2} \to \frac{\pi}{3}, \frac{5\pi}{3}$

$\cos x \neq 0$

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**۲**

$5\underbrace{\sin^2 x}_{1-\cos^2 x} + 2\underbrace{\cos(3x)}_{4\cos^3 x - 3\cos x} = -2 \to 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

تعداد اعضای ازاینه که دامنه

$\times \leftarrow 0$

$\times \leftarrow \frac{\pi}{2}$

$-\pi, \pi$ ✓✓ ۲ تا جواب $\leftarrow [-\pi, \pi]$ سمت دو بازه $\checkmark \leftarrow \pi$

$\times \leftarrow \frac{3\pi}{2}$

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**۳**

$2\sin x \cos^2 x + \sin x = 1 \to \sin x(2\underbrace{\cos^2 x}_{1-2\sin^2 x} + 1) = 1 \Rightarrow \sin x(3 - 4\sin^2 x) = 1$

$3\sin x - 4\sin^3 x = 1 \to \sin 3x = 1 \to 3x = 2k\pi + \frac{\pi}{2} \to x = \frac{2k\pi}{3} + \frac{\pi}{6}$

$\frac{3\pi}{2} \leftarrow \checkmark \quad k = 2$ $\quad\quad \times \quad k = -1$

$\times \quad k = 3$ $\quad\quad \frac{\pi}{6} \leftarrow \checkmark \quad k = 0$

$\frac{5\pi}{6} \leftarrow \checkmark \quad k = 1$

$\underbrace{\frac{\pi}{6} + \frac{5\pi}{6}}_{\pi} + \frac{3\pi}{2} = \frac{5\pi}{2}$

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**۴**

$\frac{1}{\sin\left(\frac{\pi+4x}{2}\right)} + \frac{1}{\cos\left(\frac{\pi+8x}{2}\right)} = 0 \to \frac{1}{\cos 2x} + \frac{1}{-2\sin 2x\cos 2x} = \frac{1}{\cos 2x}\left(\frac{2\sin 2x - 1}{2\sin 2x}\right) = 0$

$\sin\left(\frac{\pi+4x}{2}\right) = \sin\left(2x + \frac{\pi}{2}\right) = \cos 2x$

$\cos\left(\frac{\pi+8x}{2}\right) = \cos\left(4x + \frac{\pi}{2}\right) = -\sin 4x = -2\sin 2x\cos 2x$

$2\sin 2x = 1 \to \sin 2x = \frac{1}{2}$ $\to 2x = \frac{\pi}{6} \to x = \frac{\pi}{12}$ ; $2x = \frac{5\pi}{6} \to x = \frac{5\pi}{12}$

اختلاف $= \frac{\pi}{3} = \alpha \to \tan 2\alpha = \tan\frac{2\pi}{3} = -\sqrt{3}$

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**۵**

$\cos x - \sin x = -(\sin x - \cos x) = -\sqrt{2}\sin\left(x - \frac{\pi}{4}\right) = \sqrt{2}\sin\left(\frac{\pi}{4} - x\right) = \sqrt{2}\cos\left(\frac{\pi}{4} + x\right) = \sqrt{2} \times \frac{1}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$

$\cos\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}} \to \cos m = \frac{1}{\sqrt{3}} \to \cos 2m = 2\cos^2 m - 1 \to 2 \times \frac{1}{3} - 1 = -\frac{1}{3} \to \cos\left(2x + \frac{\pi}{2}\right) = -\frac{1}{3}$

جای گذاری $\to m \times \frac{\sqrt{2}}{\sqrt{3}} - 3\sqrt{6} \times \frac{1}{3} = \sqrt{6} \to m = \frac{2\sqrt{6} \times \sqrt{3}}{\cancel{\sqrt{2}}} = \boxed{6}$

$\boxed{6}$

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$\sin(x+\frac{\pi}{4})\cos(\frac{\pi}{4}-x)=1 \to \cos^2(\frac{\pi}{4}-x)=1$

با هم متمم

$\cos(x-\frac{\pi}{4})=\pm1 \to x-\frac{\pi}{4}=k\pi \to x=k\pi+\frac{\pi}{4} \longrightarrow \{\frac{\pi}{4},\frac{5\pi}{4}\}$ ۶

ابتدا ...

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$\sin x\times\frac{1}{2}+\cos x\times\frac{\sqrt{3}}{2}=\frac{\sqrt{2}}{2} \to \sin x\cos\frac{\pi}{3}+\sin\frac{\pi}{3}\cos x=\sin\frac{\pi}{4}$

$\sin(x+\frac{\pi}{3})=\sin(\frac{\pi}{4})$

$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \to x=2k\pi-\frac{\pi}{12} \xrightarrow{\text{در بازه}} -\frac{\pi}{12},\ \frac{23\pi}{12}$ ۷

$x+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4} \to x=2k\pi+\frac{5\pi}{12} \xrightarrow{\text{در بازه}} \frac{5\pi}{12}$

$\frac{9\pi}{4}=\frac{27\pi}{12} \in$ جمع

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$4\sin x\sin(\frac{3\pi}{2}+x)=1 \longrightarrow -4\sin x\cos x=1 \to -2\sin 2x=1$

$-\cos x$

$\sin 2x=-\frac{1}{2}$

$\sin 2x=\sin(-\frac{\pi}{6})$

$2x=2k\pi-\frac{\pi}{6} \to x=k\pi-\frac{\pi}{12} \Rightarrow$

$2x=2k\pi+\frac{7\pi}{6} \to x=k\pi+\frac{7\pi}{12} \Rightarrow$ ۸

$x=\frac{11\pi}{12},\ \frac{23\pi}{12}$

$x=\frac{7\pi}{12},\ \frac{19\pi}{12}$

$\frac{11\pi}{12}+\frac{23\pi}{12}+\frac{7\pi}{12}+\frac{19\pi}{12}=5\pi$

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$\underbrace{(1+\cos 2\alpha)}_{2\cos^2\alpha}\underbrace{(1+\cos 4\alpha)}_{2\cos^2 2\alpha}\underbrace{(1+\cos 8\alpha)}_{2\cos^2 4\alpha}=\frac{1}{8}\times\frac{\sin^2\alpha}{\sin^2\alpha} \quad \frac{8}{\sin^2\alpha}\Big(\underbrace{\underbrace{\sin^2\alpha\cos^2\alpha}_{\frac{1}{4}\sin^2 2\alpha}\cos^2 2\alpha\cos^2 4\alpha}_{\frac{1}{64}\sin^2 8\alpha}\Big)=\frac{1}{8}$

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$\frac{8}{\sin^2\alpha}\times\frac{1}{64}\times\sin^2 8\alpha=\frac{1}{8} \to \sin^2 8\alpha=\sin^2\alpha \to \sin 8\alpha=\sin\alpha$ ۹

$8\alpha=2k\pi+\alpha \to \alpha=\frac{2k\pi}{7}$

$8\alpha=2k\pi+\pi-\alpha \to \alpha=\frac{2k\pi}{9}+\frac{\pi}{9}$

$\sin 8\alpha=\sin(-\alpha)$

$8\alpha=2k\pi-\alpha \to \alpha=\frac{2k\pi}{9}$

$8\alpha=2k\pi+\pi+\alpha \to \alpha=\frac{2k\pi}{7}+\frac{\pi}{7}$

$\alpha \longrightarrow$ کمترین مثبت $=\frac{\pi}{9}$

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$\tan x=\frac{1}{\tan 3x}=\cot 3x \to \tan(\frac{\pi}{2}-3x)$

(۱,۵)

$\Rightarrow \tan x=\tan(\frac{\pi}{2}-3x)$ ۱۰

$x=k\pi+\frac{\pi}{2}-3x \to x=\frac{k\pi}{4}+\frac{\pi}{8} \xrightarrow{\text{در بازه}} \frac{5\pi}{8}+\frac{7\pi}{8}+\frac{12\pi}{8}=\frac{3\pi}{2}$

| k | ۴ | ۵ | ۶ | ۷ |
|---|---|---|---|---|
| x | $\frac{9\pi}{8}$ | $\frac{11\pi}{8}$ | $\frac{13\pi}{8}$ | $\frac{15\pi}{8}$ |

$(\frac{9+11+13+15}{8})\pi=\frac{48\pi}{8}=6\pi$

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$$\tan 3x = \frac{1}{\tan x} = \cot x \longrightarrow \tan 3x = \tan\left(\frac{\pi}{2} - x\right)$$

$$3x = k\pi + \frac{\pi}{2} - x \longrightarrow 4x = k\pi + \frac{\pi}{2} \longrightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$$

| $k$ | 4 | 5 | 6 | 7 |
|---|---|---|---|---|
| $x$ | $\frac{9\pi}{8}$ | $\frac{11\pi}{8}$ | $\frac{13\pi}{8}$ | $\frac{15\pi}{8}$ |

$$S = \left(\frac{9+11+13+15}{8}\right)\pi = \frac{48\pi}{8} = \boxed{6\pi}$$