نام و نام خانوادگی ..ستایش.......مهرزاده..... پاسخنامه تشریحی تکلیف شماره ...۱۷.. کلاس ..دوازدهم ریاضی A.........

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$8\cos x - \tan^2 x = 1 \longrightarrow 8\cos x = 1 + \tan^2 x \longrightarrow 8\cos x = \frac{1}{\cos^2 x}$

$1 + \tan^2 x = 1 + \frac{\sin^2}{\cos^2} = \frac{1}{\cos^2 x}$

$\cos^3 x = \frac{1}{8}$

$\cos x = \frac{1}{2} \longrightarrow \frac{\pi}{3}, \frac{5\pi}{3}$

$\cos x \neq 0$

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$\underbrace{5\sin^2 x}_{1-\cos^2 x} + \underbrace{2\cos(3x)}_{4\cos^3 x - 3\cos x} = -2 \longrightarrow 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

تمامی عامل‌های اول ۷ کدام

$0 \to \times$

$\frac{\pi}{2} \to \times$

$\pi \to \checkmark$ ... در بازه $[-\pi, \pi]$ ← ۲ جواب $-\pi, \pi$ ✓✓

$\frac{3\pi}{2} \to \times$

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$2\sin x\cos 2x + \sin x = 1 \longrightarrow \sin x(2\underbrace{\cos 2x}_{1-2\sin^2 x} + 1) = 1 \Rightarrow \sin x(3 - 4\sin^2 x) = 1$

$3\sin x - 4\sin^3 x = 1 \longrightarrow \sin 3x = 1 \longrightarrow 3x = 2k\pi + \frac{\pi}{2} \longrightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6}$

$k = -1 \quad \times$

$k = 0 \quad \checkmark \to \frac{\pi}{6}$

$k = 1 \quad \checkmark \to \frac{5\pi}{6}$

$k = 2 \quad \checkmark \to \frac{3\pi}{2}$

$k = 3 \quad \times$

$\underbrace{\frac{\pi}{6} + \frac{5\pi}{6}}_{\pi} + \frac{3\pi}{2} = \frac{5\pi}{2}$

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$\frac{1}{\sin\left(\frac{\pi+4x}{2}\right)} + \frac{1}{\cos\left(\frac{\pi+8x}{2}\right)} = 0 \longrightarrow \frac{1}{\cos 2x} + \frac{1}{-2\sin 2x\cos 2x} = \frac{1}{\cos 2x}\left(\frac{2\sin 2x - 1}{2\sin 2x}\right) = 0$

$\sin\left(\frac{\pi+4x}{2}\right) = \sin\left(2x + \frac{\pi}{2}\right) = \cos 2x$

$\cos\left(\frac{\pi+8x}{2}\right) = \cos\left(4x + \frac{\pi}{2}\right) = -\sin 4x = -2\sin 2x\cos 2x$

$2\sin 2x = 1 \longrightarrow \sin 2x = \frac{1}{2} \longrightarrow 2x = \frac{\pi}{6} \longrightarrow x = \frac{\pi}{12}$

$\qquad\qquad\qquad\qquad\qquad\qquad 2x = \frac{5\pi}{6} \longrightarrow x = \frac{5\pi}{12}$

اختلاف $= \frac{\pi}{3} = \alpha \longrightarrow \tan 2\alpha = \tan\frac{2\pi}{3} = -\sqrt{3}$

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$\cos x - \sin x = -(\sin x - \cos x) = -\sqrt{2}\sin\left(x - \frac{\pi}{4}\right) = \sqrt{2}\sin\left(\frac{\pi}{4} - x\right) = \sqrt{2}\cos\left(\frac{\pi}{4} + x\right) \Rightarrow \sqrt{2} \times \frac{1}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$

$\cos\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}} \longrightarrow \cos m = \frac{1}{\sqrt{3}} \longrightarrow \cos 2m = 2\cos^2 m - 1 \longrightarrow 2 \times \frac{1}{3} - 1 = -\frac{1}{3} \Rightarrow \cos\left(2x + \frac{\pi}{2}\right) = -\frac{1}{3}$

$\xrightarrow{\text{جایگذاری}} m \times \frac{\sqrt{2}}{\sqrt{3}} - 3\sqrt{6} \times \frac{1}{3} = \sqrt{6} \longrightarrow m = \frac{\cancel{2\sqrt{6}} \times \sqrt{3}}{\cancel{\sqrt{2}}} = \boxed{6}$

$\sin\left(x+\frac{\pi}{6}\right)\cos\left(\frac{\pi}{3}-x\right)=1 \longrightarrow \cos^2\left(\frac{\pi}{3}-x\right)=1$ ۶

اتحاد متمم

$\cos\left(x-\frac{\pi}{3}\right)=\pm1 \longrightarrow x-\frac{\pi}{3}=k\pi \longrightarrow x=k\pi+\frac{\pi}{3} \xrightarrow{[0,2\pi]} \left\{\frac{\pi}{3},\ \frac{4\pi}{3}\right\}$

$\sin x\cdot\frac{1}{2}+\cos x\cdot\frac{\sqrt{3}}{2}=\frac{\sqrt{2}}{2} \longrightarrow \sin x\cos\frac{\pi}{3}+\sin\frac{\pi}{3}\cos x=\sin\frac{\pi}{4}$ ۷

$\sin\left(x+\frac{\pi}{3}\right)=\sin\left(\frac{\pi}{4}\right)$

- $x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \longrightarrow x=2k\pi-\frac{\pi}{12} \xrightarrow{\text{[illegible]}} -\frac{\pi}{12},\ \frac{23\pi}{12}$
- $x+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4} \longrightarrow x=2k\pi+\frac{5\pi}{12} \xrightarrow{\text{[illegible]}} \frac{5\pi}{12}$

$\frac{9\pi}{4}=\frac{27\pi}{12} \in$ جمع

$4\sin x\,\sin\left(\frac{3\pi}{2}+x\right)=1 \longrightarrow -4\sin x\cos x=1 \longrightarrow -2\sin^2 x=1$

$\sin\left(\frac{3\pi}{2}+x\right)=-\cos x$

$\sin 2x=-\frac{1}{2}$

$\sin 2x=-\frac{\pi}{6}$

- $2x=2k\pi-\frac{\pi}{6} \longrightarrow x=k\pi-\frac{\pi}{12} \Rightarrow$
- $2x=2k\pi+\frac{7\pi}{6} \longrightarrow x=k\pi+\frac{7\pi}{12} \Rightarrow$

$x=\frac{11\pi}{12}$ و $\frac{23\pi}{12}$

$x=\frac{7\pi}{12}$ و $\frac{19\pi}{12}$

$\frac{11\pi}{12}+\frac{23\pi}{12}+\frac{7\pi}{12}+\frac{19\pi}{12}=5\pi$

$\underbrace{(1+\cos 2\alpha)}_{2\cos^2\alpha}\underbrace{(1+\cos 4\alpha)}_{2\cos^2 2\alpha}\underbrace{(1+\cos 8\alpha)}_{2\cos^2 4\alpha}=\frac{1}{8} \xrightarrow{\times \sin^2\alpha} \frac{8}{\sin^2\alpha}\left(\underbrace{\underbrace{\sin^2\alpha\cos^2\alpha}_{\frac{1}{4}\sin^2 2\alpha}\cos^2 2\alpha\cos^2 4\alpha}_{\frac{1}{64}\sin^2 8\alpha}\right)=\frac{1}{8}$ ۹

$\frac{8}{\sin^2\alpha}\times\frac{1}{64}\times\sin^2 8\alpha=\frac{1}{8} \longrightarrow \sin^2 8\alpha=\sin^2\alpha \longrightarrow$

- $\sin 8\alpha=\sin\alpha$
  - $8\alpha=2k\pi+\alpha \longrightarrow \alpha=\frac{2k\pi}{7}$
  - $8\alpha=2k\pi+\pi-\alpha \longrightarrow \alpha=\frac{2k\pi}{9}+\frac{\pi}{9}$
- $\sin 8\alpha=\sin(-\alpha)$
  - $8\alpha=2k\pi-\alpha \longrightarrow \alpha=\frac{2k\pi}{9}$
  - $8\alpha=2k\pi+\pi+\alpha \longrightarrow \alpha=\frac{2k\pi}{7}+\frac{\pi}{7}$

$\alpha \longrightarrow$ [illegible] $=\frac{8\pi}{9}$

$\tan x=\frac{1}{\tan 3x}=\cot 3x=\tan\left(\frac{\pi}{2}-3x\right)$ ۱۰

$\Rightarrow \tan x=\tan\left(\frac{\pi}{2}-3x\right)$

$x=k\pi+\frac{\pi}{2}-3x \longrightarrow x=\frac{k\pi}{4}+\frac{\pi}{8} \xrightarrow{\text{[illegible]}} \frac{5\pi}{8}+\frac{7\pi}{8}=\frac{12\pi}{8}=\frac{3\pi}{2}$