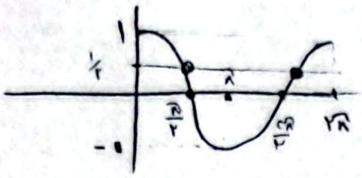
رها ارشدی — ۱۰۷۵ — پاسخنامه را با دقت مطالعه کنید.

①

$8\cos x = 1+\tan^2 x \longrightarrow \left(8\cos x = \frac{1}{\cos^2 x}\right)\times \cos^2 x$

$8\cos^3 x = 1 \longrightarrow \cos^3 x = \frac{1}{8} \longrightarrow \cos x = \frac{1}{2}$

۲



$2k\pi + \frac{\pi}{3}$

$2k\pi - \frac{\pi}{3}$

۲تا جواب دارد.

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②

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$\cos 3x = 2\cos^2\frac{3x}{2} - 1$ (فرمول طلایی)

$5\sin^2 x + 2\left(2\cos^2\frac{3x}{2} - 1\right) = -2 \longrightarrow 5\sin^2 x + 4\cos^2\frac{3x}{2} \cancel{-2} = \cancel{-2}$

حاصل جمع دو عدد نامنفی صفر است پس هر دو صفر بوده اند!

$5\sin^2 x = 0 \longrightarrow \sin x = 0 \longrightarrow x = k\pi \xrightarrow{-\pi\le x\le \pi} x = \pi$ یا $-\pi$ یا 0

$4\cos^2\frac{3x}{2} = 0 \longrightarrow \cos\frac{3x}{2} = 0 \longrightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} \longrightarrow x = \frac{2k\pi+\pi}{3}$

$\xrightarrow{x\in[-\pi,\pi]} x = -\pi$ یا $-\frac{\pi}{3}$ یا $\frac{\pi}{3}$ یا π

اشتراک دو مجموعه جواب $\{\pi, -\pi\}$ است

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③

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$2\sin x(\cos^2 x - \sin^2 x) + \sin x = 1$ $\qquad \cos^2 + \sin^2 = 1$

$2\sin x(1 - 2\sin^2 x) + \sin x = 1$

$2\sin x - 4\sin^3 x + \sin x = 1 \longrightarrow 3\sin x - 4\sin^3 x = 1$

$\sin 3x = 1 \longrightarrow$

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④

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$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} \longrightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0$

$\frac{1}{\cos 2x} = \frac{1}{\sin 4x} \Rightarrow \sin 4x = \cos 2x$

$2\sin 2x\cos 2x = \cos 2x \xrightarrow{\cos 2x\neq 0} \sin 2x = \frac{1}{2}$

$\sin 2x = \sin\frac{\pi}{6}$

$2x = 2k\pi + \frac{\pi}{6} \Rightarrow x = k\pi + \frac{\pi}{12}$

$2x = 2k\pi + \pi - \frac{\pi}{6} \Rightarrow x = k\pi + \frac{5\pi}{12}$

$\Rightarrow \frac{\pi}{12}, \frac{5\pi}{12}$

$\alpha = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3} \longrightarrow \tan 2\alpha \longrightarrow \tan\frac{2\pi}{3} \Rightarrow \tan\left(\pi - \frac{\pi}{3}\right)$

$\Rightarrow -\tan\frac{\pi}{3} = -\sqrt{3}$

۵

$\cos(x+\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \rightarrow \cos x - \sin x = \frac{\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}}$

$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \rightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$

$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \rightarrow \boxed{\sin 2x = \frac{1}{3}}$

$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \rightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}(\frac{1}{3}) = \sqrt{6}$

$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \rightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \rightarrow \boxed{m=6}$

۶

$\sin(x+\frac{\pi}{6})\cos(x-\frac{\pi}{3}) = 1$ $\qquad \frac{\pi}{6}+\frac{\pi}{3} = \frac{9\pi}{18} = \frac{\pi}{2} \Rightarrow$ متمم یکدیگر هستند.

$\Rightarrow \cos(\frac{\pi}{3}-x)\cos(\frac{\pi}{3}-x) = 1 \Rightarrow \cos^2(x-\frac{\pi}{3}) = 1$

$x - \frac{\pi}{3} = k\pi \longrightarrow x = k\pi + \frac{\pi}{3} \xrightarrow{[0,2\pi]} x = \frac{\pi}{3} \,/\, \frac{4\pi}{3}$

۷

عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$

$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2}$

$\begin{cases} x+\frac{\pi}{3} = 2k\pi+\frac{\pi}{4} \rightarrow x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi\le x\le 2\pi} k=0,1 \rightarrow x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x+\frac{\pi}{3} = 2k\pi+\frac{3\pi}{4} \rightarrow x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi\le x\le 2\pi} k=0 \rightarrow x = \frac{5\pi}{12} \end{cases}$

$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$

۸

$\sin(\frac{5\pi}{2} - x) = -\cos x$

$2x = 2k\pi + (-\frac{\pi}{6}) \Rightarrow x = k\pi - \frac{\pi}{12}$

$\longrightarrow x = \frac{11\pi}{12}, \frac{23\pi}{12}$

$2x = 2k\pi + \pi - (-\frac{\pi}{6}) \Rightarrow x = k\pi + \frac{7\pi}{12} \longrightarrow x = \frac{7\pi}{12} \,|\, \frac{19\pi}{12}$

$\frac{11\pi}{12} + \frac{23\pi}{12} + \frac{7\pi}{12} + \frac{19\pi}{12} = \frac{60\pi}{12} = 5\pi$

⑨

۱٫۷۵

$$(2\cos^2\frac{\alpha}{2})\times(2\cos^2\alpha)\times(2\cos^2 2\alpha)=\frac{1}{8}$$

$$(\cos\frac{\alpha}{2}\cos\alpha\cos 2\alpha)^2=\frac{1}{64}\longrightarrow\cos\frac{\alpha}{2}\cos\alpha\cos 2\alpha=\pm\frac{1}{8}$$

$$\xrightarrow{\times 8\sin\frac{\alpha}{2}}\underbrace{8\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}_{\frac{1}{2}\sin\alpha}\cos\alpha\cos 2\alpha=\pm\sin\frac{\alpha}{2}$$

$$\Rightarrow\underbrace{4\sin\alpha\cos\alpha}_{\sin 2\alpha}2\cos 2\alpha=\pm\sin\frac{\alpha}{2}\Rightarrow\sin 4\alpha=\pm\sin\frac{\alpha}{2}$$

حالت ۱ اگر $\sin 4\alpha=\sin\frac{\alpha}{2}$

$$4\alpha=2k\pi+\frac{\alpha}{2}\Rightarrow\alpha=\frac{4}{7}k\pi$$
$$4\alpha=2k\pi+\pi-\frac{\alpha}{2}\Rightarrow\alpha=\frac{4}{9}k\pi+\frac{2\pi}{9}$$

حالت ۲ اگر $\sin 4\alpha=\sin(-\frac{\alpha}{2})$

$$4\alpha=2k\pi-\frac{\alpha}{2}\Rightarrow\alpha=\frac{4k\pi}{9}$$
$$4\alpha=2k\pi+\pi+\frac{\alpha}{2}\Rightarrow\alpha=\frac{4}{7}k\pi+\frac{2\pi}{7}$$

$$\xrightarrow{\alpha\in[0,2\pi]}\alpha=\frac{4\pi}{7}\,|\,\frac{8\pi}{7}\,|\,\frac{12\pi}{7}\,|\,\frac{4\pi}{9}\,|\,\frac{8\pi}{9}\,|\,\frac{12\pi}{9}\,|\,\frac{16\pi}{9}\,|\,\frac{2\pi}{7}\,|\,\frac{6\pi}{7}\,|\,\frac{10\pi}{7}\,|\,\frac{2\pi}{9}\,|\,\frac{10\pi}{9}\,|\,\frac{14\pi}{9}$$

مجموعاً ۱۴ جواب دارد. MANA !

⑩ ۲

$$\tan 3x=\frac{1}{\tan x}=\cot x\longrightarrow\tan 3x=\tan(\frac{\pi}{2}-x)$$

$$3x=k\pi+\frac{\pi}{2}-x\longrightarrow 4x=k\pi+\frac{\pi}{2}\rightarrow x=\frac{k\pi}{4}+\frac{\pi}{8}$$

k	4	5	6	7
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$$S=(\frac{9+11+13+15}{8})\pi=\frac{48\pi}{8}=\boxed{6\pi}$$

(۳)

$\cos 2x = 1 - 2\sin^2 x$ (فرمول طلایی)

۳ ۴ ۳

$2\sin x(1 - 2\sin^2 x) + \sin x = 1 \longrightarrow 3\sin x - 4\sin^3 x = 1$

$\sin 3x = 1 \longrightarrow 3x = 2k\pi + \frac{\pi}{2} \longrightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \xrightarrow{0 \leq x \leq 2\pi}$

$k=0 \longrightarrow x = \frac{\pi}{6}\ ,\ k=1 \longrightarrow x = \frac{5\pi}{6}\ ,\ k=2 \longrightarrow x = \frac{9\pi}{6}$

$S = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$

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(۹)

$1 + \cos 2x = 2\cos^2 x$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \overset{?}{\longrightarrow} 8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{8}$$

$\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\cos 2\alpha\cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha \star}$

$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\cos 4\alpha = \pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$

$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\cos 4\alpha)}_{\sin 8\alpha} = \frac{\pm\sin\alpha}{8} \longrightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \frac{\pm\sin\alpha}{\cancel{8}}$

$$\sin 8\alpha = \pm\sin\alpha \longrightarrow \begin{cases} 8\alpha = 2k\pi + \alpha \ \text{یا}\ 2k\pi + \pi - \alpha \\ 8\alpha = 2k\pi - \alpha \ \text{یا}\ 2k\pi + \pi + \alpha \end{cases}$$

$\alpha = \frac{2k\pi}{7} \xrightarrow{MAX} k=3 \longrightarrow x = \frac{6\pi}{7}$

$\alpha = \frac{2k\pi}{9} \xrightarrow{MAX} k=4 \longrightarrow x = \frac{8\pi}{9}$

$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{MAX} k=2 \longrightarrow x = \frac{5\pi}{7}$

$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{MAX} k=3 \longrightarrow x = \frac{7\pi}{9}$

$\longrightarrow \boxed{MAX = \frac{8\pi}{9}}$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ sinα را مخالف صفر در نظر گرفته‌ایم!