

$$\sin\left(x + \frac{\pi}{2}\right) \cos\left(x - \frac{\pi}{2}\right) = 1 \xrightarrow[\text{نیز از این معادله می توانیم:}]{\cos\left(x - \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2} - x\right)} \sin\left(x + \frac{\pi}{2}\right) \cdot \sin\left(x + \frac{\pi}{2}\right) = 1 \Rightarrow \sin^2\left(x + \frac{\pi}{2}\right) = 1$$

فرض کنیم $x + \frac{\pi}{2} = \alpha$

$$\Rightarrow \sin\left(x + \frac{\pi}{2}\right) = \begin{cases} 1 \\ -1 \end{cases} \Rightarrow \begin{cases} x + \frac{\pi}{2} = \frac{\pi}{2} \\ x + \frac{\pi}{2} = \frac{3\pi}{2} \end{cases} \Rightarrow \begin{cases} x = 0 \\ x = \pi \end{cases}$$

$\Rightarrow$ جواب: $\frac{\pi}{2}$ و $\frac{3\pi}{2}$

$$\sin x + \sqrt{2} \cos x = \sqrt{2} \Rightarrow \frac{1}{\sqrt{2}} \sin(x + \alpha) = \sqrt{2} \Rightarrow \sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\tan \alpha = \sqrt{2} \Rightarrow \alpha = \frac{\pi}{4}$$

$$\Rightarrow \begin{cases} x + \frac{\pi}{4} = 2k\pi + \frac{\pi}{4} \Rightarrow x = 2k\pi \\ x + \frac{\pi}{4} = 2k\pi + \frac{3\pi}{4} \Rightarrow x = 2k\pi + \frac{\pi}{2} \end{cases}$$



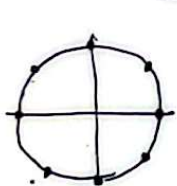
$\rightarrow [-\pi, \pi]$ جواب: $\frac{\pi}{4}$ و $\frac{3\pi}{4}$

$$4 \sin x \cdot \sin\left(\frac{3\pi}{4} - x\right) = 1 \xrightarrow[\text{با استفاده از فرمول}]{\sin(A-B) = \sin A \cos B - \cos A \sin B} 4 \sin x \cos x = 1 \Rightarrow \sin 2x = \frac{1}{4}$$

$$\Rightarrow \begin{cases} 2x = \frac{\pi}{4} \Rightarrow x = \frac{\pi}{8} \\ 2x = \frac{7\pi}{4} \Rightarrow x = \frac{7\pi}{8} \end{cases} \rightarrow \text{مجموع: } \frac{1 \cdot \pi}{12} = \frac{\pi}{12}$$

$$(1 + \cos 2x)(1 + \cos 4x)(1 + \cos 8x) = \frac{1}{8} \xrightarrow[\text{با استفاده از فرمول}]{\text{طبق فرمول}} (\frac{2 \cos^2 x}{2})(\frac{2 \cos^2 2x}{2})(\frac{2 \cos^2 4x}{2}) = \frac{1}{8}$$

$$\xrightarrow{\times \sin^2 x} 2 \cos^2 x (\sin^2 x) (\cos^2 2x) (\cos^2 4x) = \frac{1}{8} \sin^2 x \Rightarrow \frac{1}{8} \sin^2 4x = \frac{1}{8} \sin^2 x \Rightarrow \sin^2 4x = \sin^2 x$$



$$\sin^2 4x = \sin^2 x \Rightarrow \frac{1}{8} \sin^2 4x = \frac{1}{8} \sin^2 x \Rightarrow \frac{1}{8} \sin^2 4x = \frac{1}{8} \sin^2 x$$

$$\Rightarrow |\sin 4x| = |\sin x|$$

$$\Rightarrow \begin{cases} \sin 4x = \sin x \Rightarrow \begin{cases} 4x = 2k\pi + x \Rightarrow x = \frac{2k\pi}{3} \\ 4x = 2k\pi + \pi - x \Rightarrow x = \frac{(2k+1)\pi}{5} \end{cases} \\ \sin 4x = -\sin x \Rightarrow \begin{cases} 4x = 2k\pi - x \Rightarrow x = \frac{2k\pi}{5} \\ 4x = 2k\pi + \pi - x \Rightarrow x = \frac{(2k+1)\pi}{3} \end{cases} \end{cases}$$

$$\tan 3x \cdot \tan x = 1 \Rightarrow \tan^2 x = \frac{1}{\tan^2 x} \Rightarrow \tan^2 x = \cot^2 x \Rightarrow \tan^2 x = \tan^2\left(\frac{\pi}{2} - x\right)$$

$$\Rightarrow 3x = k\pi + \frac{\pi}{2} - x \Rightarrow 4x = k\pi + \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$$



$$\text{جواب ها: } \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8} \rightarrow \text{مجموع: } \frac{\pi}{2}$$