۱) $8\cos x = \tan^2 x + 1 \rightarrow 8\cos x = \frac{1}{\cos^2 x} \rightarrow \cos^3 x = \frac{1}{8} \rightarrow$

$\cos x = \frac{1}{2} \rightarrow x = 2k\pi \pm \frac{\pi}{3}$ ← در بازه‌ی مشخص شده ۲ جواب داره

۲) $5\sin^2 x + 2\cos^3 x + 2 = 0 \rightarrow 5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) + 2 = 0$

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0 \rightarrow \cos x = -1$

$(\cos x + 1)(8\cos^2 x - 12\cos x + 7) = 0 \rightarrow \cos x = -1 \rightarrow x = 2k\pi - \pi$

$8\cos^2 x - 12\cos x + 7 < 0$

← ۲ جواب دارد

۳) $\sin x(2\cos 2x + 1) = 1 \rightarrow \sin x(2 - 4\sin^2 x + 1) = 1 \rightarrow$

$4\sin^3 x - 3\sin x + 1 = 0 \rightarrow (\sin x + 1)(4\sin^2 x - 4\sin x + 1) = 0$

$(\sin x + 1)(2\sin x - 1)^2 = 0 \rightarrow \sin x = \{-1, \frac{1}{2}\} \rightarrow$

$x = \{2k\pi - \frac{\pi}{2}, 2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}\} \rightarrow$ در بازه‌ی مشخص شده،

جواب های $\frac{\pi}{6}$ ، $\frac{5\pi}{6}$ ، $\frac{3\pi}{2}$ قابل قبول اند ← مجموع $= \frac{9\pi}{2}$

۴) $\frac{1}{\sin(2x + \frac{\pi}{2})} + \frac{1}{\cos(4x + \frac{\pi}{2})} = 0 \rightarrow \frac{1}{\cos 2x} + \frac{-1}{\sin 4x} = 0 \rightarrow$

$\frac{1}{\cos 2x} = \frac{1}{\sin 4x} \rightarrow \cos 2x = \sin 4x \rightarrow \cos^2 x = 2\sin 2x \cos 2x \rightarrow \sin 2x = \frac{1}{2}$

$2x = \{2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}\} \rightarrow x = \{k\pi + \frac{\pi}{12}, k\pi + \frac{5\pi}{12}\}$

← ۲ جواب در بازه‌ی مشخص شده $\frac{\pi}{12}, \frac{5\pi}{12}$ ← $\alpha = \frac{\pi}{3}$

$\tan \frac{2\pi}{3} = -\sqrt{3}$

$\cos\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}} \rightarrow \frac{\sqrt{2}}{2}(\cos x-\sin x)=\frac{1}{\sqrt{3}} \rightarrow$ (۵

$\cos x-\sin x=\frac{\sqrt{2}}{\sqrt{3}}$

$(\cos x-\sin x)^2=1-\sin 2x \rightarrow \sin 2x=1-\frac{2}{3}=\frac{1}{3}$

$\frac{\sqrt{2}}{\sqrt{3}}(m)-2\sqrt{6}\left(\frac{1}{3}\right)=\sqrt{3} \rightarrow \frac{\sqrt{2}}{\sqrt{3}}m=2\sqrt{6} \rightarrow m=6$

$\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{3}\right)=1 \rightarrow$ (۶

$2\cos\left(x-\frac{\pi}{3}\right)=1 \rightarrow \cos\left(x-\frac{\pi}{3}\right)=\frac{1}{2} \rightarrow$

$x=\frac{\pi}{3}=2k\pi \pm \frac{\pi}{3} \rightarrow x=\left\{2k\pi+\frac{2\pi}{3}, 2k\pi\right\}$

تعداد جواب ها بازه‌ی مشخص شده ۳ تا هست

$\sin x+\frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}\cos x=\sqrt{2} \rightarrow$ (۷

$\frac{\cos\frac{\pi}{3}\sin x+\sin\frac{\pi}{3}\cos x}{\cos\frac{\pi}{3}}=\sqrt{2} \rightarrow \sin\left(x+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2} \rightarrow$

$x+\frac{\pi}{3}=\left\{2k\pi+\frac{\pi}{4}, 2k\pi+\frac{3\pi}{4}\right\} \rightarrow$

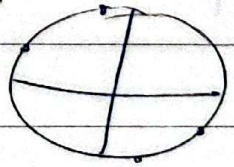
$x=\left\{2k\pi-\frac{\pi}{12}, 2k\pi+\frac{5\pi}{12}\right\} \rightarrow$ جواب‌ها: $\left\{-\frac{\pi}{12}, \frac{23\pi}{12}, \frac{5\pi}{12}\right\}$

تعداد $=\frac{9\pi}{4}$

$4\sin x \times \sin\left(\frac{3\pi}{2}-x\right)=1 \rightarrow -2\sin 2x=1 \rightarrow$ (۸

$\sin 2x=-\frac{1}{2} \rightarrow 2x=\left\{2k\pi-\frac{\pi}{6}, 2k\pi-\frac{5\pi}{6}\right\} \rightarrow$

$x=\left\{k\pi-\frac{\pi}{12}, k\pi-\frac{5\pi}{12}\right\}$



تعداد جواب در بازه‌ی مشخص شده

$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha) = \frac{1}{8} \longrightarrow$ (۹

$(2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha) = \frac{1}{8} \longrightarrow$

$\sin 8\alpha = 8 \sin\alpha \cos\alpha \cos 2\alpha \cos 4\alpha \longrightarrow$

$\frac{|\sin 8\alpha|}{|\sin\alpha|} = 1 \longrightarrow \sin 8\alpha = \sin\alpha \;\; \text{یا} \;\; \sin 8\alpha = \sin -\alpha \longrightarrow$

$\alpha = \left\{ \frac{2k\pi}{9}, \frac{2k\pi}{7}, \frac{2k\pi+\pi}{7}, \frac{2k\pi+\pi}{9} \right\} \longrightarrow$

راه‌حل دوم:

$\alpha = \left\{ \frac{\pi}{7}, \frac{2\pi}{7}, \frac{3\pi}{7}, \frac{\pi}{7}, \frac{4\pi}{7}, \frac{5\pi}{7}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{6\pi}{9}, \frac{8\pi}{9}, \right.$

$\left. \frac{\pi}{2}, \frac{3\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9} \right\}$

$\max \alpha = \frac{8\pi}{9}$

$\tan^2 x \times \tan x = 1 \longrightarrow \tan^2 x = \cot x \longrightarrow$ (۱۰

$\tan^2 x = \tan\left(\frac{\pi}{2} - x\right) \longrightarrow 2x = k\pi + \frac{\pi}{2} - x \longrightarrow$

$3x = k\pi + \frac{\pi}{2} \longrightarrow x = \frac{k\pi}{3} + \frac{\pi}{6} \longrightarrow$

راه‌حل دوم: $x = \left\{ \frac{9\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{15\pi}{6} \right\} \longrightarrow$ مجموع $= 9\pi$