(سارا امینی‌راد)

14

$8\cos x - \frac{\sin^2 x}{\cos^2 x} = 1 \to 8\cos x - \frac{1-\cos^2 x}{\cos^2 x} = 1 \to 8\cos^3 x - 1 = 0$

① $\cos^3 x = \frac{1}{8} \to \cos x = \frac{1}{2}$

یا $\frac{\pi}{3}, \frac{5\pi}{3} \leftarrow$ ۲ جواب

2

② $5\sin^2 x + 2\cos(2x) = -2 \to 5\sin^2 x + 2\cos 2x + 2 = 0$ $x = \pi$, $x = -\pi$ $\to$ تعداد جواب $= 2$

2

③ $\sin x(2\cos 2x + 1) = 1 \to \sin x(2(1-2\sin^2 x)+1) = \sin x(3 - 4\sin^2 x) = 1$

$\sin x = t \to -4t^3 + 3t - 1 = 0 \to 4t^3 - 3t + 1 = 0 \to t = -1 \to \sin x = -1 \to x = \frac{3\pi}{2}$

ادامه؟

1

④ $\frac{6x+\pi}{2} = 3x + \frac{\pi}{2}$ $\sin(3x + \frac{\pi}{2}) = \cos(3x)$ $\sin 4x = \cos 2x$

$\frac{8x+\pi}{2} = 4x + \frac{\pi}{2}$ $\cos(4x + \frac{\pi}{2}) = -\sin(4x)$ $\sin 4x = \sin(\frac{\pi}{2} - 2x)$

$\to x = \frac{\pi}{12} + \frac{k\pi}{3}$ $x = \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{3\pi}{4} \to$ تعریف نشده

$\to x = \frac{\pi}{4} + k\pi$ $x = \frac{3\pi}{4} \to \tan(3\pi) = 0$

1,5

⑤ $m(\cos x - \sin x) - 2\sqrt{6}\sin 2x = \sqrt{6}$

$\cos(x + \frac{\pi}{4}) = \frac{1}{\sqrt{3}} \to \frac{\cos x - \sin x}{\sqrt{2}} = \frac{1}{\sqrt{3}} \to \cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}}$

$(\cos x - \sin x)^2 = 1 - \sin(2x) \to \frac{2}{3} = 1 - \sin 2x \to \sin 2x = \frac{1}{3}$

$m\sqrt{\frac{2}{3}} - 2\sqrt{6}(\frac{1}{3}) = \sqrt{6} \to m\sqrt{\frac{2}{3}} = 2\sqrt{6} \to m = 6$

2

⑥ $\sin(x + \frac{\pi}{4})\cos(x - \frac{\pi}{3}) = 1$

$\sin(x + \frac{\pi}{4}) = \pm 1 \to \sin\theta = 1 \to 2k\pi + \frac{\pi}{2}$ $\sin\theta = -1 \to 2k\pi + \frac{3\pi}{2}$

$x + \frac{\pi}{4} = \frac{\pi}{2} + 2k\pi$ $x + \frac{\pi}{4} = 2k\pi + \frac{3\pi}{2} \to x = 2k\pi + \frac{5\pi}{4}$

$\cos(x - \frac{\pi}{3}) = \pm 1$ $x = 2k\pi + \frac{\pi}{4}$

$\cos\theta = 1$ $\cos x = -1 \to x - \frac{\pi}{3} = \pi + 2k\pi$

$x - \frac{\pi}{3} = 2k\pi$ $x = 2k\pi + \frac{4\pi}{3}$

$x = \frac{\pi}{3} + 2k\pi$

1,5

⑦ $\sin x + \sqrt{3}\cos x = 2\sin(x + \frac{\pi}{3}) = \sqrt{2} \to \sin(x + \frac{\pi}{3}) = \frac{\sqrt{2}}{2}$

$x + \frac{\pi}{3} = \frac{\pi}{4} + 2k\pi$ $x = 2k\pi - \frac{\pi}{12}$

$x + \frac{\pi}{3} = \frac{3\pi}{4} + 2k\pi$ $x = 2k\pi + \frac{5\pi}{12}$ $x_1 = -\frac{\pi}{12} / \frac{5\pi}{12} / \frac{23\pi}{12}$

$\to$ جمع $\frac{9\pi}{4}$

2

(سارا امینی راد)

⑧

$4\sin x\sin(\frac{3\pi}{2}-x)=1 \rightarrow \sin(\frac{3\pi}{2}-x)=\sin\frac{3\pi}{2}\cos x-\cos\frac{3\pi}{2}\sin x=-\cos x$

$4\sin x(-\cos x)=1 \rightarrow -4\sin x\cos x=1 \rightarrow -2\sin 2x=1 \rightarrow \sin 2x=-\frac{1}{2}$

$2x=2k\pi-\frac{\pi}{6} \rightarrow x=k\pi-\frac{\pi}{12}$

$2x=2k\pi+\frac{7\pi}{6} \rightarrow x=k\pi+\frac{7\pi}{12}$

$\rightarrow x\in\{\frac{7\pi}{12},\frac{11\pi}{12},\frac{19\pi}{12},\frac{23\pi}{12}\} \rightarrow$ جمع $=\frac{60\pi}{12}=5\pi$

۲

⑨

⑩

$\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}\times\tan x=1 \rightarrow \tan^4 x-6\underbrace{\tan^2 x}_{t}+1=0$

$\rightarrow t^2-6t+1=0 \rightarrow \tan^2 x=3+2\sqrt{2}$ ، $\tan^2 x=3-2\sqrt{2}$

$\rightarrow \tan x=\tan(\frac{\pi}{8}) \rightarrow x=\frac{\pi}{8}+\frac{k\pi}{4}$

$\{\frac{9\pi}{8},\frac{11\pi}{8},\frac{13\pi}{8},\frac{15\pi}{8}\} \rightarrow \frac{48\pi}{8}=6\pi$

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③

$\cos 2x=1-2\sin^2 x$ (فرمول طلایی)

$2\sin x(1-2\sin^2 x)+\sin x=1 \rightarrow 3\sin x-4\sin^3 x=1$

$\sin 3x=1 \rightarrow 3x=2k\pi+\frac{\pi}{2} \rightarrow x=\frac{2k\pi}{3}+\frac{\pi}{6} \xrightarrow{0\leq x\leq 2\pi}$

$k=0 \rightarrow x=\frac{\pi}{6}$, $k=1 \rightarrow x=\frac{5\pi}{6}$, $k=2 \rightarrow x=\frac{9\pi}{6}$

$S=\frac{\pi}{6}+\frac{5\pi}{6}+\frac{9\pi}{6}=\frac{15\pi}{6}=\boxed{\frac{5\pi}{2}}$

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④

$\begin{cases}\sin(2x+\frac{\pi}{2})=\cos 2x\\ \cos(\frac{\pi}{2}+4x)=-\sin 4x\end{cases} \rightarrow \frac{1}{\cos 2x}+\frac{1}{-\sin 4x}=0 \rightarrow \frac{\cos 2x-\sin 4x}{\cos 2x(-\sin 4x)}=0$

$\cos 2x-2\sin 2x\cos 2x=0 \rightarrow \cos 2x(1-2\sin 2x)=0 \rightarrow \cos 2x=0$ ✗ ، $\sin 2x=\frac{1}{2}$ ✓

* اگر $\cos 2x=0$ باشد مخرج عبارت اولیه صفر می‌شود که غیر قابل قبول است!

$\sin 2x=\frac{1}{2} \rightarrow 2x=2k\pi+\frac{\pi}{6}$ یا $2k\pi+\frac{5\pi}{6} \rightarrow x=k\pi+\frac{\pi}{12}$ یا $k\pi+\frac{5\pi}{12}$

$\xrightarrow{0\leq x\leq\pi} k=0 \rightarrow x=\frac{\pi}{12}$ یا $\frac{5\pi}{12} \rightarrow \frac{5\pi}{12}-\frac{\pi}{12}=\frac{\pi}{3}$

$\alpha=\frac{\pi}{3} \rightarrow \tan(2\alpha)=\tan(\frac{2\pi}{3})=\boxed{-\sqrt{3}}$

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$$\cos\left(x-\frac{\pi}{3}\right)=\cos\left(\frac{\pi}{3}-x\right)$$

$$\frac{\pi}{3}-x+x+\frac{\pi}{6}=\frac{\pi}{2} \longrightarrow \cos\left(\frac{\pi}{3}-x\right)=\sin\left(x+\frac{\pi}{6}\right)$$

$$\sin^2\left(x+\frac{\pi}{6}\right)=1 \longrightarrow \sin\left(x+\frac{\pi}{6}\right)=\pm 1 \longrightarrow x+\frac{\pi}{6}=k\pi+\frac{\pi}{2}$$

$$x=k\pi+\frac{\pi}{3} \xrightarrow{0\leq x\leq 2\pi} k=0 \to x=\frac{\pi}{3}\ ,\quad k=1\ ,\ x=\frac{4\pi}{3}$$

معادله ۲ جواب در بازه دارد

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(۹)

$$1+\cos 2* = 2\cos^2 *$$

$$\left.\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases}\right\} ? \to 8\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha=\frac{1}{8}$$

$$\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha=\frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\cos 2\alpha\cos 4\alpha=\frac{\pm 1}{8} \xrightarrow[\sin\alpha\neq 0]{\times \sin\alpha\ *}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\cos 4\alpha=\pm\frac{\sin\alpha}{8} \to \frac{1}{2}\underbrace{(\sin 2\alpha\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha=\pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\cos 4\alpha)}_{\sin 8\alpha}=\frac{\pm\sin\alpha}{8} \longrightarrow \frac{1}{\cancel{8}}\sin 8\alpha=\pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha=\pm\sin\alpha \longrightarrow \begin{cases} 8\alpha=2k\pi+\alpha \ \text{ یا } \ 2k\pi+\pi-\alpha \\ 8\alpha=2k\pi-\alpha \ \text{ یا } \ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha=\frac{2k\pi}{7} \xrightarrow{\max A} k=3 \to x=\frac{6\pi}{7}$$

$$\alpha=\frac{2k\pi}{9} \xrightarrow{\max A} k=4 \to x=\frac{8\pi}{9}$$

$$\alpha=\frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \to x=\frac{5\pi}{7} \quad \longrightarrow \boxed{\max A=\frac{8\pi}{9}}$$

$$\alpha=\frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \to x=\frac{7\pi}{9}$$

* A نمی‌تواند برابر خود π باشد چون طبق * sinα را مخالف صفر در نظر گرفته‌ایم!