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حل تست محوری ، دوازدهم تجربی

۱) : $8\cos x - \tan^2 x = 1$ ، ساده شده عبارت صورت سوال

$\to -\frac{1}{\cos^2 x} + 8\cos x + \cancel{1} = \cancel{1} \Rightarrow \frac{-1}{\cos^2 x} + 8\cos x = 0$ ۲

$\Rightarrow 8\cos x = \frac{1}{\cos^2 x} \Rightarrow 8\cos^3 x = 1 \Rightarrow \cos^3 x = \frac{1}{8}$

$\Rightarrow \cos x = \frac{1}{2} \Rightarrow$ (1) $x = 2k\pi + \frac{\pi}{3}$

(2) $x = 2k\pi + \frac{5\pi}{3}$

۲) جواب دارد

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۲) : (ب)

$\cos 3x = 2\cos^2\frac{3x}{2} - 1$ (فرمول طلایی)

$5\sin^2 x + 2\left(2\cos^2\frac{3x}{2} - 1\right) = -2 \to 5\sin^2 x + 4\cos^2\frac{3x}{2} - \cancel{2} = \cancel{-2}$

حاصل جمع دو عدد نامنفی صفر است پس هر دو صفر بوده اند!

$5\sin^2 x = 0 \to \sin x = 0 \to x = k\pi \xrightarrow{-\pi \le x \le \pi} x = \pi$ یا $-\pi$ یا 0

$4\cos^2\frac{3x}{2} = 0 \to \cos\frac{3x}{2} = 0 \to \frac{3x}{2} = k\pi + \frac{\pi}{2} \to x = \frac{2k\pi + \pi}{3}$

$\xrightarrow{x \le \pi} x = -\pi$ یا $-\frac{\pi}{3}$ یا $\frac{\pi}{3}$ یا π

$\boxed{\text{اشتراک دو مجموعه جواب } \{\pi, -\pi\} \text{ است}}$

③ :

$$2\sin(x)\cos(2x)+\sin(x)=1$$

$\cos^2 x-\sin^2 x$

$1-\sin^2 x$

②

$$\Rightarrow 2\sin(x)(1-2\sin^2 x)+\sin(x)=1$$

$$\Rightarrow 2\sin x-4\sin^3 x+\sin(x)=1 \Rightarrow -4\sin^3 x+3\sin x=1$$

$\Rightarrow$ ① $x=\frac{\pi}{6}$, ② $\frac{5\pi}{6}$ ② جواب دارد

④ : ساده شده‌ی عبارت صورت سوال $\Rightarrow \frac{1}{\cos 2x}+\frac{1}{-\sin 4x}=0$

②

~~شهادت مظلومانه آیت‌الله ... بهشتی و ۷۲ تن از یاران امام خمینی(ره) (۱۳۶۰ ش)~~

$$\Rightarrow \frac{1}{\cos 2x}=\frac{1}{\sin 4x} \Rightarrow \sin 4x=\cos 2x$$

$$2\sin 2x\cos 2x=\cos 2x \xrightarrow{\cos 2x\neq 0} \sin 2x=\frac{1}{2}$$

$$\Rightarrow \sin 2x=\sin\frac{\pi}{6} \begin{cases} 2x=2k\pi+\frac{\pi}{6} \Rightarrow x=k\pi+\frac{\pi}{12} \\ 2x=2k\pi+\pi-\frac{\pi}{6} \Rightarrow x=k\pi+\frac{5\pi}{12} \end{cases}$$

$$\Rightarrow \frac{\pi}{12} , \frac{5\pi}{12}$$

$$\alpha=\frac{5\pi}{12}-\frac{\pi}{12}=\frac{\pi}{3} \rightarrow \tan 2\alpha \rightarrow \tan\frac{2\pi}{3}=-\sqrt{3}$$

۵

$$\cos\left(x+\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \longrightarrow \cos x - \sin x = \frac{\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \longrightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \longrightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \longrightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}\left(\frac{1}{3}\right) = \sqrt{6}$$

$$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \longrightarrow \boxed{m=6}$$

۶

$$\cos\left(x-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}-x\right)$$

$$\frac{\pi}{3} - x + x + \frac{\pi}{6} = \frac{\pi}{2} \longrightarrow \cos\left(\frac{\pi}{3}-x\right) = \sin\left(x+\frac{\pi}{6}\right)$$

$$\sin^2\left(x+\frac{\pi}{6}\right) = 1 \longrightarrow \sin\left(x+\frac{\pi}{6}\right) = \pm 1 \longrightarrow x + \frac{\pi}{6} = k\pi + \frac{\pi}{2}$$

$$x = k\pi + \frac{\pi}{3} \xrightarrow{0 \le x \le 2\pi} k=0 \rightarrow x = \frac{\pi}{3}, \quad k=1, \ x = \frac{4\pi}{3}$$

معادله ۲ جواب در بازه دارد

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۷ عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0,1 \rightarrow x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \longrightarrow x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0 \rightarrow x = \frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

8

$$\sin\left(\frac{3\pi}{2}-x\right) = -\cos x$$

$$4\sin x(-\cos x)=1 \longrightarrow -2(2\sin x\cos x)=1 \longrightarrow \sin 2x = -\frac{1}{2}$$

$$2x = 2k\pi - \frac{\pi}{6} \text{ یا } 2k\pi + \frac{7\pi}{6} \longrightarrow x = \overset{2}{k\pi - \frac{\pi}{12}} \text{ یا } \overset{1}{k\pi + \frac{7\pi}{12}}$$

$$0 \le x \le 2\pi \longrightarrow \begin{cases} 1: & k=0,1 \longrightarrow x = \frac{7\pi}{12}, \frac{19\pi}{12} \\ 2: & k=1,2 \longrightarrow x = \frac{11\pi}{12}, \frac{23\pi}{12} \end{cases}$$

$$S = \frac{7\pi}{12} + \frac{19\pi}{12} + \frac{11\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = \boxed{5\pi}$$

10

$$\tan 3x = \frac{1}{\tan x} = \cot x \longrightarrow \tan 3x = \tan\left(\frac{\pi}{2} - x\right)$$

$$3x = k\pi + \frac{\pi}{2} - x \longrightarrow 4x = k\pi + \frac{\pi}{2} \longrightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$$

k	4	5	6	7
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$$S = \left(\frac{9+11+13+15}{8}\right)\pi = \frac{48\pi}{8} = \boxed{6\pi}$$

۹

$$1+\cos 2x = 2\cos^2 x$$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \rightarrow 8\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha \cos 2\alpha \cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha \ \bigstar}$$

$$\underbrace{(\sin\alpha \cos\alpha)}_{\sin 2\alpha} \cos 2\alpha \cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha \cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha} \cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha \cos 4\alpha)}_{\sin 8\alpha} = \frac{\pm\sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا} \ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا} \ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \rightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \rightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \rightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \rightarrow x = \frac{7\pi}{9}$$

$$\rightarrow \boxed{\max A = \frac{8\pi}{9}}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!