

① $\Delta \cos x - \tan^2 x = 1 \quad [0, 2\pi]$

ریشه های

$$\Delta \cos x = 1 + \tan^2 x \rightarrow \Delta \cos x = \frac{1}{\cos^2 x} \rightarrow \Delta \cos^2 x = 1 \rightarrow \cos^2 x = \frac{1}{2}$$



② $\Delta \sin^2 x + 2 \cos^2 x = -2$

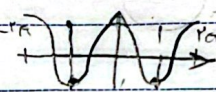
$$\Delta \sin^2 x + 2(1 - \sin^2 x) = -2 \rightarrow \Delta(1 - \cos^2 x) + 2\cos^2 x - 2 = -2$$

$$\Delta - \Delta \cos^2 x + 2\cos^2 x - 2 = -2 \xrightarrow{\cos x = a} \Delta - \Delta a^2 + 2a^2 - 2 = -2$$

$$\Delta - \Delta a^2 + 2a^2 - 2 = -2 \xrightarrow{\Delta = a+1} \Delta a^2 - \Delta a^2 - 4a + 2a^2 = 0 \xrightarrow{\Delta = a+1} (a+1)(\Delta a^2 - 4a + 2a^2) = 0$$

$$\Delta a^2 - \Delta a^2 - 4a + 2a^2 = 0 \xrightarrow{\Delta = a+1} -1a^2 - 4a + 2a^2 = 0 \xrightarrow{\Delta = a+1} a^2 - 4a = 0 \xrightarrow{\Delta = a+1} a(a-4) = 0$$

$$a = 0 \text{ or } a = 4$$



ریشه های

③ $2 \sin x \cos^2 x + \sin x = 1 \quad [0, 2\pi]$

$$2 \sin^2 x = 1 - \cos^2 x$$

$$\sin x(2 \cos^2 x + 1) = 1 \rightarrow \sin x(2(1 - \sin^2 x) + 1) = 1 \rightarrow \sin x(2 - 2 \sin^2 x + 1) = 1$$

$$2 \sin x - 2 \sin^3 x - 1 = 0$$

$$2 \sin^3 x - 2 \sin x + 1 = 0 \quad \Delta = \frac{4 - 16}{4} = -3$$

ریشه های

④ $\frac{1}{\sin(\frac{\pi}{4} + \frac{t}{2})} + \frac{1}{\cos(\frac{\pi}{4} + \frac{t}{2})} = 0 \quad x_1 - x_2 = \alpha \quad [0, \alpha] \quad \tan \alpha = ?$

$$\frac{1}{\sin(\frac{\pi}{4} + \frac{t}{2})} + \frac{1}{\cos(\frac{\pi}{4} + \frac{t}{2})} = 0 \rightarrow \frac{1}{\cos^2 t} + \frac{1}{-\sin^2 t} = 0 \rightarrow \frac{-\sin^2 t + \cos^2 t}{-\sin^2 t \cos^2 t} = 0$$



$$\cos^2 t - \sin^2 t = 0 \rightarrow \sin^2 t = \cos^2 t \rightarrow \sin t = \cos t \rightarrow \sin t = \sin(\frac{\pi}{4} - t) \rightarrow t = \frac{\pi}{4} - t \rightarrow t = \frac{\pi}{8}$$

$$t = \frac{\pi}{8} \rightarrow x_1 = \frac{\pi}{4} + \frac{t}{2} = \frac{\pi}{4} + \frac{\pi}{16} = \frac{5\pi}{16}$$

$$x_2 = \frac{\pi}{4} + \frac{t}{2} = \frac{\pi}{4} + \frac{\pi}{16} = \frac{5\pi}{16}$$

$$x_1 - x_2 = \frac{\pi}{8} \rightarrow \tan \frac{\pi}{8} = \frac{1 - \cos \frac{\pi}{4}}{\sin \frac{\pi}{4}} = \frac{1 - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2 - \sqrt{2}}{\sqrt{2}} = \sqrt{2} - 1$$

⑤ $m(\cos x - \sin x) - 2\sqrt{2} \sin^2 x = \sqrt{2}$ $\cos x \cos \frac{\pi}{4} - \sin x \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$\frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{2}} \rightarrow \cos x - \sin x = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$m(\frac{\sqrt{2}}{2}) - 2\sqrt{2} \sin^2 x = \sqrt{2} \rightarrow \frac{m}{2} - 2 \sin^2 x = 1 \rightarrow \frac{m}{2} = 1 + 2 \sin^2 x$$

$$m = 2 + 4 \sin^2 x$$

$$(4) \sin\left(n + \frac{\pi}{4}\right) \cos\left(n - \frac{\pi}{4}\right) = 1 \quad [0, 2\pi]$$

$$\left(\sin n \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos n\right) \left(\cos n \cos \frac{\pi}{4} + \sin n \cos \frac{\pi}{4}\right) = 1$$

$$\left(\frac{\sqrt{2}}{2} \sin n + \frac{1}{2} \cos n\right) \left(\frac{1}{2} \cos n + \frac{\sqrt{2}}{2} \sin n\right) = 1$$

$$\rightarrow \sin\left(n + \frac{\pi}{4}\right) = 1 \quad x + \frac{\pi}{4} = 2K\pi + \frac{\pi}{4} \rightarrow y_n + \alpha = 2K\pi + \frac{\pi}{4} \rightarrow y_n = 2K\pi + \frac{\pi}{4}$$

$$\rightarrow n = \frac{2K\pi}{4}$$

$$\rightarrow \sin\left(n + \frac{\pi}{4}\right) = -1 \quad x + \frac{\pi}{4} = 2K\pi - \frac{\pi}{4} \rightarrow y_n + \alpha = 2K\pi - \frac{\pi}{4} \rightarrow y_n = 2K\pi - \frac{\pi}{4}$$

$$n = \frac{2K\pi}{4}$$



$$[-\pi, \pi] (5) \sin n + \sqrt{2} \cos n = \sqrt{2} \quad r\left(\frac{1}{r} \sin n + \sqrt{2} \cos n\right) = \sqrt{2} \rightarrow \sin\left(n + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{r}$$

$$\rightarrow x + \frac{\pi}{4} = 2K\pi + \frac{\pi}{4} \quad x = 2K\pi - \frac{\pi}{4} \rightarrow n = \frac{2K\pi - \pi}{4}$$

$$\text{or } \frac{-\pi}{4} \quad \frac{2K\pi}{4}$$

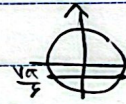
$$\rightarrow x + \frac{\pi}{4} = 2K\pi + \frac{\pi}{4} \quad x = 2K\pi + \frac{\pi}{4} \rightarrow n = 2K\pi + \frac{\pi}{4}$$

$$\frac{-19\pi}{4} \quad \frac{\Delta\pi}{4}$$

$$\frac{\Delta\pi}{4} - \frac{\pi}{4} + \frac{2K\pi}{4} - \frac{19\pi}{4} = \frac{\Delta\pi}{4}$$

$$\frac{2K\pi}{4} = \frac{\Delta\pi}{4}$$

$$(6) \sin n \cos n = 1 \rightarrow \sin 2n = 1$$



$$\rightarrow 2n = 2K\pi + \frac{\pi}{2} \rightarrow n = K\pi + \frac{\pi}{4}$$

$$(7) (\cos n)(\cos n)(\cos n) = \frac{1}{\sqrt{2}}$$

$$\frac{\cos n}{\sqrt{2}} = \frac{1}{\sqrt{2}} \rightarrow \cos n = 1 \rightarrow n = 2K\pi \rightarrow n = 2K\pi$$

$$\rightarrow \cos n = -1 \rightarrow n = 2K\pi + \pi \rightarrow n = (2K+1)\pi$$

$$\rightarrow \sin n = 1 \rightarrow n = 2K\pi + \frac{\pi}{2} \rightarrow n = 2K\pi + \frac{\pi}{2}$$

$$\rightarrow \sin n = -1 \rightarrow n = 2K\pi + \frac{3\pi}{2} \rightarrow n = 2K\pi + \frac{3\pi}{2}$$

$$\text{Doodle: } \frac{\Delta\pi}{4}$$

$$(8) \tan n = \cot n \rightarrow \tan n = \tan\left(\frac{\pi}{2} - n\right) \quad n = K\pi + \frac{\pi}{4} \rightarrow n = \frac{\pi}{4}(2K+1)$$

$$\frac{\Delta\pi}{4} \quad \frac{5\pi}{4} \quad \frac{9\pi}{4} \quad \frac{13\pi}{4} \quad \frac{17\pi}{4} \quad \frac{21\pi}{4}$$

$$\frac{2K\pi}{4} = \frac{\Delta\pi}{4}$$