①

$$8\cos x - \tan^2 x = 1 \Rightarrow 8\cos x = 1+\tan^2 x \Rightarrow 8\cos x = \frac{1}{\cos^2 x} \Rightarrow 8\cos^3 x = 1$$

$$\Rightarrow \cos^3 x = \frac{1}{8} \Rightarrow \cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3} \Rightarrow$$ $\boxed{\text{در بازه‌ی گفته شده ۲ جواب دارد}}$

②

$$5\sin^2 x + 2\cos 3x + 2 = 0 \Rightarrow 5\sin^2 x + 4\cos^2\left(\frac{3x}{2}\right) = 0 \Rightarrow$$ جمع دو عبارت همواره نامنفی هستند صفر شده است پس هر دو صفر هستند ⇐

$$5\sin^2 x = 0 \Rightarrow \sin x = 0$$

$$\Rightarrow x = k\pi \xrightarrow{\text{در بازه‌ی } [-\pi,\pi]} x = (-\pi), 0, \pi \quad ①$$

$$4\cos^2\left(\frac{3x}{2}\right) = 0 \Rightarrow \cos\frac{3x}{2} = 0 \Rightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} \Rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{3} \quad ②$$

$$\xrightarrow[\text{در بازه‌ی } [-\pi,\pi]]{① \cap ②} x = (-\pi), \pi \Rightarrow$$ $\boxed{\text{۲ جواب دارد}}$

③

$$2\sin x\cos 2x + \sin x - 1 = 0 \Rightarrow 2\sin x(1-2\sin^2 x) + \sin x - 1 = 0$$

$$\Rightarrow 3\sin x - 4\sin^3 x - 1 = 0 \Rightarrow 4\sin^3 x - 3\sin x + 1 = 0$$

$$\Rightarrow (\sin x + 1)(4\sin^2 x - 4\sin x + 1) = 0$$

$$\Rightarrow \begin{cases} \sin x = -1 \Rightarrow x = \frac{3\pi}{2} \\ (2\sin x - 1)^2 = 0 \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6} \end{cases} \Rightarrow \frac{3\pi}{2} + \frac{\pi}{6} + \frac{5\pi}{6} = \boxed{\frac{5\pi}{2}}$$

④

$$\sin\left(\frac{\pi}{2} + 2x\right) = -\cos\left(\frac{\pi}{2} + 4x\right) \Rightarrow \cos 2x = \sin 4x \Rightarrow$$ با توجه به صورت سوال

چون این دو در مخرج قرار دارند پس $\sin 4x$ و $\cos 2x$ نمی‌توانند صفر باشند.

$$\Rightarrow \sin 4x - \cos 2x = 0 \Rightarrow 2\sin 2x\cos 2x - \cos 2x = 0 \Rightarrow \cos 2x(2\sin 2x - 1) = 0$$

$$\Rightarrow \begin{cases} \cos 2x = 0 \times \text{غ ق ق} \\ \sin 2x = \frac{1}{2} \xrightarrow{\text{در بازه‌ی } [0,\pi]} 2x = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12} \end{cases}$$

$$\Rightarrow \alpha = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3} \Rightarrow \boxed{\tan 2\alpha = \tan\frac{2\pi}{3} = -\sqrt{3}}$$

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DATE :

رونیا ابوئی

۵)

$$\cos\left(x+\frac{\pi}{4}\right)=\cos x\cos\frac{\pi}{4}-\sin x\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x$$

$$=\frac{\sqrt{2}}{2}(\cos x-\sin x)=\frac{1}{\sqrt{3}}\Rightarrow \cos x-\sin x=\frac{\sqrt{2}}{\sqrt{3}}$$

$$\xrightarrow{\text{توان ۲}}\sin^2x+\cos^2x-2\sin x\cos x=\frac{2}{3}\Rightarrow 1-\sin 2x=\frac{2}{3}$$

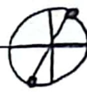
$$\Rightarrow \sin 2x=\frac{1}{3}\Rightarrow m(\cos x-\sin x)-3\sqrt{6}\sin(2x)=\sqrt{6}$$

$$\Rightarrow m\left(\frac{\sqrt{2}}{\sqrt{3}}\right)-3\sqrt{6}\left(\frac{1}{3}\right)=\sqrt{6}\Rightarrow \frac{m\sqrt{6}}{3}=2\sqrt{6}\Rightarrow \boxed{m=6}$$

۶)

$$\sin\left(x+\frac{\pi}{6}\right)\cos\left(\frac{\pi}{3}-x\right)=1\Rightarrow \sin\left(x+\frac{\pi}{6}\right)\times\sin\left(x+\frac{\pi}{6}\right)=1$$

$$\Rightarrow \sin^2\left(x+\frac{\pi}{6}\right)=1\Rightarrow \sin\left(x+\frac{\pi}{6}\right)=\pm 1\Rightarrow x+\frac{\pi}{6}=k\pi+\frac{\pi}{2}$$

$\Rightarrow x=k\pi+\frac{\pi}{3}\Rightarrow$  $\rightarrow \frac{\pi}{3}\,,\,\frac{4\pi}{3}\Rightarrow$ $\boxed{\text{۲ جواب دارد}}$ در بازه‌ی خواسته شده

۷)

$$\sqrt{3}=\tan\frac{\pi}{3}$$

$$\sin x+\frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}\times\cos x=\sqrt{2}\Rightarrow \frac{\sin x\cos\frac{\pi}{3}+\sin\frac{\pi}{3}\cos x}{\cos\frac{\pi}{3}}=\sqrt{2}$$

$$\Rightarrow \sin\left(x+\frac{\pi}{3}\right)=\sqrt{2}\cos\frac{\pi}{3}=\frac{\sqrt{2}}{2}\Rightarrow x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4}\,,\,2k\pi+\frac{3\pi}{4}$$

$$\Rightarrow x=\frac{-\pi}{12}\,,\,\frac{5\pi}{12}\,,\,\frac{23\pi}{12}\xrightarrow[\text{در بازه‌ی }[-\pi,2\pi]]{\text{جمع جواب ها}}\boxed{\frac{9\pi}{4}}$$

۸)

$$4\sin x\overbrace{\sin\left(\frac{3\pi}{2}-x\right)}^{-\cos x}=-4\sin x\cos x=-2\sin 2x=1$$

$$\Rightarrow \sin 2x=-\frac{1}{2}=\sin\left(-\frac{\pi}{6}\right)\Rightarrow\begin{cases}2x=2k\pi-\frac{\pi}{6}\Rightarrow x=k\pi-\frac{\pi}{12}\\[4pt] 2x=2k\pi+\frac{7\pi}{6}\Rightarrow x=k\pi+\frac{7\pi}{12}\end{cases}$$

$$\xrightarrow[\text{در بازه‌ی }[0,2\pi]]{\text{جمع جواب ها}}\frac{7\pi}{12}+\frac{11\pi}{12}+\frac{19\pi}{12}+\frac{23\pi}{12}=\boxed{5\pi}$$

رونیا ایوبی

9

$$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\frac{1}{8}$$

$$\Rightarrow (2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)=\frac{1}{8}\Rightarrow(\cos^2\alpha)(\cos^2 2\alpha)(\cos^2 4\alpha)=\frac{1}{64}$$

$$\Rightarrow(\cos\alpha)(\cos 2\alpha)(\cos 4\alpha)=\pm\frac{1}{8}\xrightarrow[\text{طرفین ضرب در}]{\times\frac{\sin\alpha}{\sin\alpha}}\frac{1}{8}\sin 8\alpha=\pm\frac{1}{8}\sin\alpha$$

$\sin\alpha\neq 0$

$$\Rightarrow \sin 8\alpha=\pm\sin\alpha=\sin(\pm\alpha)$$

$$\xrightarrow{\sin 8\alpha=\sin\alpha}\begin{cases}8\alpha=2k\pi+\alpha\Rightarrow\alpha=\frac{2k\pi}{7}\\ 8\alpha=2k\pi+\pi-\alpha\Rightarrow\alpha=\frac{2k\pi}{9}+\frac{\pi}{9}\end{cases}$$

$$\xrightarrow{\sin 8\alpha=\sin(-\alpha)}\begin{cases}8\alpha=2k\pi-\alpha\Rightarrow\alpha=\frac{2k\pi}{9}\\ 8\alpha=2k\pi+\pi+\alpha\Rightarrow\alpha=\frac{2k\pi}{7}+\frac{\pi}{7}\end{cases}$$

$\Rightarrow$ ماکسیمم عضو جواب معادله (ماکسیمم عضو مجموعه A) در بازه ی $[0,\pi]$ $\Leftarrow \boxed{\frac{8\pi}{9}}$

10

$$\tan(3x)\tan(x)=1\Rightarrow\tan 3x=\frac{1}{\tan x}\Rightarrow\tan 3x=\cot x$$

$$\Rightarrow\tan 3x=\tan\left(\frac{\pi}{2}-x\right)\Rightarrow 3x=k\pi+\frac{\pi}{2}-x$$

$$\Rightarrow 4x=k\pi+\frac{\pi}{2}\Rightarrow x=\frac{k\pi}{4}+\frac{\pi}{8}$$

مجموع جواب های معادله در بازه ی $[\pi,2\pi]$:

$$\frac{9\pi}{8}+\frac{11\pi}{8}+\frac{13\pi}{8}+\frac{15\pi}{8}=\boxed{6\pi}$$