Subject :
Year . Month . Date .

لطفا واضح تر عکس بگیرید! ۱۸/۵

نیایش زندی

(۱)

$8\cos x = \tan^2 x + 1 \qquad \frac{1}{\cos^2 x} = 8\cos x \qquad 8\cos^3 x = 1 \qquad \cos^3 x = \frac{1}{8}$

$\cos x = \frac{1}{2} \longrightarrow 2k\pi \pm \frac{\pi}{3} \xrightarrow[{[0,2\pi]}]{\text{در بازه}} \left\{ \frac{\pi}{3} , \frac{5\pi}{3} \right\}$ ۲

(۲)

$5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) = -2$

$5 - 5\cos^2 x + 8\cos^3 x - 6\cos x + 2 = 0 \longrightarrow 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

جمع ضرایب زوج و فرد ها برابر است یکی از ریشه ها ۱- است. ۲

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 \;\big|\; \cos x + 1$

$-8\cos^3 x \pm 8\cos^2 x \qquad 8\cos^2 x - 13\cos x + 7 \qquad 8\cos^2 x - 13\cos x + 7 = 0$

$-13\cos^2 x - 6\cos x + 7$

$-13\cos^2 x \mp 13\cos x$

$7\cos x + 1$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{13 \pm \sqrt{169 - 4(\ldots)}}{16}$

$\Delta < 0$ ⇐ در نتیجه این معادله ریشه ندارد و تنها جواب $\cos x = -1$ است.

$\longleftarrow 2k\pi + \pi$ ← بالا در بازه $[-\pi , \pi]$

$\{-\pi , \pi\} \rightarrow$ ۲ تا

(۳)

$\sin x + (\sin 3x - \sin x) = 1 \qquad \sin 3x = 1 \qquad 3x = 2k\pi + \frac{\pi}{2}$ ۲

$x = \frac{\pi}{6} + \frac{2k\pi}{3} \xrightarrow[\text{در بازه خواسته شده}]{\text{جواب های}} \left\{ \frac{\pi}{6} , \frac{5\pi}{6} , \frac{3\pi}{2} \right\} \xrightarrow{\text{مجموع}} \frac{5\pi}{2}$

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(4) — ۱,۵

$$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = 0 \rightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0$$

$$\frac{-\sin 4x + \cos 2x}{\cos 2x \, \sin 4x} = 0 \qquad \cos 2x - \sin 4x = 0 \qquad \cos 2x = \sin 4x$$

$$\cos 2x = \cos(\frac{3\pi}{2} + 4x) \qquad \sin 4x = \sin(\frac{\pi}{2} - 2x)$$

$$4x = \frac{\pi}{2} - 2x + 2k\pi \longrightarrow x = \frac{k\pi}{3} + \frac{\pi}{12} \longrightarrow \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{3\pi}{4}$$

$$4x = \pi - (\frac{\pi}{2} - 2x) + 2k\pi \longrightarrow x = k\pi + \frac{\pi}{4}$$

اختلاف بزرگترین و کوچکترین جواب $= \frac{2\pi}{3}$

$$2a = \frac{4\pi}{3} \qquad \tan(\frac{4\pi}{3}) = \sqrt{3}$$

$$\frac{5\pi}{4} - \frac{\pi}{12} = \frac{16}{12} = \frac{4\pi}{3}$$

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(5)

$$m(\cos x - \sin x) - 6\sqrt{6}\,\sin x\cos x = \sqrt{6} \qquad \cos(x+\frac{\pi}{4}) = \frac{\cos x - \sin x}{\sqrt{2}}$$

$$\cos x - \sin x = \sqrt{\frac{2}{3}} \qquad \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}} \qquad 1 - \sin 2x = \frac{2}{3}$$

$$\sin 2x = \frac{1}{3} \qquad m\left(\sqrt{\frac{2}{3}}\right) - 6\sqrt{6}\left(\frac{1}{6}\right) = \sqrt{6} \qquad m = \frac{2\sqrt{6}}{\sqrt{\frac{2}{3}}} \qquad \boxed{m = 6}$$

۲ ✓

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(6) — ۱,۵

$$\sin(x+\frac{\pi}{6}) \times \cos(x+\frac{\pi}{6}-\frac{\pi}{2}) = 1 \qquad (2)\ \sin(x+\frac{\pi}{6}) = 1$$

در عبارت در هم ضرب شده اند $\quad \sin^2(x+\frac{\pi}{6}) = 1$

$$\sin(x+\frac{\pi}{6}) = \frac{1}{2}$$

$$x+\frac{\pi}{6} = 2k\pi + \frac{\pi}{6} \qquad x = 2k\pi$$

$$x+\frac{\pi}{6} = 2k\pi + \frac{5\pi}{6} \qquad x = 2k\pi + \frac{2\pi}{3}$$

در بازه صفر تا $2\pi \leftarrow \{0, 2\pi, \frac{2\pi}{3}\} \Rightarrow$ ۳ تا

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(7)

$$\frac{a}{|a|}\sqrt{a^2+b^2}\,\sin(x+\varphi) \rightarrow \tan\varphi = \frac{b}{a} \rightarrow 2\sin(x+\frac{\pi}{3}) = \sqrt{2}$$

$$\sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2} \qquad x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \rightarrow x = 2k\pi + \frac{3\pi - 4\pi}{12} = 2k\pi - \frac{\pi}{12}$$

$$x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \rightarrow x = 2k\pi + \frac{9\pi - 4\pi}{12} = 2k\pi + \frac{5\pi}{12}$$

$$\left\{-\frac{\pi}{12}, \frac{23\pi}{12}, \frac{5\pi}{12}\right\}$$

مجموع $\rightarrow \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$ ✓

۲

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(۸)

$-2\sin 2x = 1 \qquad \sin 2x = -\frac{1}{2}$

(۲)

$2x = 2k\pi - \frac{\pi}{6} \longrightarrow x = k\pi - \frac{\pi}{12}$

$2x = 2k\pi - \frac{5\pi}{6} \qquad x = k\pi - \frac{5\pi}{12}$

$\left\{\frac{11\pi}{12}, \frac{23\pi}{12}, \frac{7\pi}{12}, \frac{19\pi}{12}\right\}$

$\frac{60\pi}{12} = 5\pi$ ✓

$(\ \ \ a-1)(1+2\cos^2 2a-1)(1+2\cos^2 4a-1) = \frac{1}{8}$

(۱,۵)

$(2\cos^2 a)(2\cos^2 2a)(2\cos^2 4a) = \frac{1}{8} \qquad \cos^2 a \cdot \cos^2 2a \cdot \cos^2 4a = \frac{1}{64}$

$(\cos a)\cdot(\cos 2a)\cdot(\cos 4a) = \frac{1}{8} \xrightarrow[\div \sin a]{\times \sin a} \left(\underbrace{\frac{1}{2}\sin 2a \cdot \cos 2a}_{\frac{1}{4}\sin 4a}\right)(\cos 4a) = \frac{1}{8}$

$\frac{1}{4}\underbrace{\sin 4a \cos 4a}_{\frac{1}{2}\sin 8a} = \frac{1}{8} \qquad \frac{1}{8}\sin 8a = \frac{\pm \sin a}{8} \qquad \sin 8a = 1 \qquad 8a = 2k\pi + \frac{\pi}{2}$

۲ طرف عبارت در $\sin a$ ضرب می‌شود

$a = \frac{k\pi}{4} + \frac{\pi}{16} \xrightarrow{\text{در بازه } [0, \pi]} \left\{\frac{\pi}{16}, \frac{5\pi}{16}, \frac{9\pi}{16}, \frac{13\pi}{16}\right\} \longrightarrow$ $\frac{13\pi}{16}$ به عنوان Max

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(۱۵)

$\tan 3x = \frac{1}{\tan x} \qquad \tan 3x = \cot x \qquad \tan 3x = -\tan\left(\frac{\pi}{2}+x\right)$

(۲)

$\tan 3x = \tan\left(-\frac{\pi}{2}-x\right) \qquad 3x = -\frac{\pi}{2} - x + k\pi \qquad 4x = k\pi - \frac{\pi}{2} \ \Big|\ x = \frac{k\pi}{4} - \frac{\pi}{8}$

مجموع $6x = \frac{48\pi}{8}$ ✓ $\longleftarrow \left\{\frac{15\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}\right\} \longleftarrow [\pi, 2\pi]$ دربازه

(۴)

$$\begin{cases}\sin(2\alpha+\frac{\pi}{2})=\cos 2\alpha\\ \cos(\frac{\pi}{2}+4\alpha)=-\sin 4\alpha\end{cases}\rightarrow \frac{1}{\cos 2\alpha}+\frac{1}{-\sin 4\alpha}=0\rightarrow\frac{\cos 2\alpha-\sin 4\alpha}{\cos 2\alpha(-\sin 4\alpha)}=0$$

$$\cos 2\alpha-2\sin 2\alpha\cos 2\alpha=0\rightarrow\cos 2\alpha(1-2\sin 2\alpha)=0$$

$\cos 2\alpha=0$ ✗

$\sin 2\alpha=\frac{1}{2}$ ✓

* اگر $\cos 2\alpha=0$ باشد مضرب عبارت اولیه صفر می‌شود که چیز قابل قبول است!

$$\sin 2\alpha=\frac{1}{2}\rightarrow 2\alpha=2k\pi+\frac{\pi}{6}\ \cup\ 2k\pi+\frac{5\pi}{6}\rightarrow\alpha=k\pi+\frac{\pi}{12}\ \cup\ k\pi+\frac{5\pi}{12}$$

$$0\le\alpha\le\pi\xrightarrow{}k=0\rightarrow\alpha=\frac{\pi}{12}\ \cup\ \frac{5\pi}{12}\rightarrow\frac{5\pi}{12}-\frac{\pi}{12}=\frac{\pi}{3}$$

$$\alpha=\frac{\pi}{3}\rightarrow\tan(2\alpha)=\tan(\frac{2\pi}{3})=\boxed{-\sqrt{3}}$$

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(۵)

$$\cos(\alpha-\frac{\pi}{2})=\cos(\frac{\pi}{2}-\alpha)$$

$$\frac{\pi}{2}-\alpha+\alpha+\frac{\pi}{4}=\frac{3\pi}{4}\rightarrow\cos(\frac{\pi}{2}-\alpha)=\sin(\alpha+\frac{\pi}{4})$$

$$\sin^2(\alpha+\frac{\pi}{4})=1\rightarrow\sin(\alpha+\frac{\pi}{4})=\pm1\rightarrow\alpha+\frac{\pi}{4}=k\pi+\frac{\pi}{2}$$

$$\alpha=k\pi+\frac{\pi}{4}\xrightarrow{0\le\alpha\le2\pi}k=0\rightarrow\alpha=\frac{\pi}{4}\ ,\ k=1\ ,\ \alpha=\frac{5\pi}{4}$$

معادله ۲ جواب در بازه دارد

$1+\cos 2\alpha = 2\cos^2\alpha$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \xrightarrow{?} 8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = 1$$

$$\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\cos 2\alpha\cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha} (*)$$

$$(\underbrace{\sin\alpha\cos\alpha}_{\frac{1}{2}\sin 2\alpha})\cos 2\alpha\cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}(\underbrace{\sin 2\alpha\cos 2\alpha}_{\frac{1}{2}\sin 4\alpha})\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}(\underbrace{\sin 4\alpha\cos 4\alpha}_{\frac{1}{2}\sin 8\alpha}) = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{8}\sin 8\alpha = \pm\frac{\sin\alpha}{8}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi + \alpha \\ 8\alpha = 2k\pi - \alpha \end{cases} \begin{cases} \alpha = \frac{2k\pi}{7} \\ \alpha = \frac{2k\pi}{9} \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{MAn\,A} k=3 \rightarrow n = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{MAn\,A} k=4 \rightarrow n = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{MAn\,A} k=2 \rightarrow n = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{MAn\,A} k=3 \rightarrow n = \frac{7\pi}{9}$$

$$\rightarrow \boxed{MAn\,A = \frac{8\pi}{9}}$$

* A نمی‌تواند برابر خود π یا 2π چون طبق $\sin\alpha$ را مضارب صفر در نظر گرفته‌ایم!