۱۸,۲۵

تکلیف ۱۷:   به نام خدا   یکتا دستانی / دوازدهم دختر A

$8\cos x = 1+\tan^2 x \longrightarrow 8\cos x = \frac{1}{\cos^2 x} \rightarrow 8\cos^3 x = 1$ ۱

$\rightarrow \cos x = \frac{1}{2} \longrightarrow \begin{cases} 2k\pi + \frac{\pi}{3} \rightarrow \frac{\pi}{3} \\ 2k\pi - \frac{\pi}{3} \rightarrow \frac{5\pi}{3} \end{cases} \rightarrow$ ۲ جواب $\rightarrow 2$

۲

$5 - 5\cos^2 + 2(4\cos^3 - 3\cos) = -2$ ۲

$\rightarrow 5 - 5\cos^2 + 8\cos^3 - 6\cos + 2 = 0 \longrightarrow 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

$\cos = -1 \longrightarrow$ جواب اصلی است

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 \;\big|\; \cos x + 1$

$-8\cos^3 x - 8\cos^2 x \qquad 8\cos^2 x - 13\cos x + 7$

$-13\cos^2 x - 6\cos x \qquad 7\cos x + 7$

$+13\cos^2 x + 13\cos x$

۲

$(\cos x+1)(8\cos^2 x - 13\cos x + 7) = 0 \longrightarrow \cos x = -1 \rightarrow$ 

$\cos = -1$ جواب ندارد $2k\pi + \pi \rightarrow \{-\pi, \pi\} \rightarrow 2$

$2\sin x(\cos^2 x - \sin^2 x) + \sin x = 1$ ۳

$2\sin x(1 - 2\sin^2 x) + \sin x = 1 \rightarrow -4\sin^3 + 3\sin = 1$

۲

$\rightarrow \sin 3x = 1 \longrightarrow$  $\rightarrow 3x = 2k\pi + \frac{\pi}{2}$

$\rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \longrightarrow x = \frac{4k\pi}{6} + \frac{\pi}{6} \rightarrow \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}\}$

$\rightarrow \frac{\pi + 5\pi + 9\pi}{6} = \frac{15\pi}{6} = \frac{5\pi}{2} \rightarrow 2$

$\sin\left(\frac{\pi}{2}+2x\right) \Rightarrow$  $\rightarrow \cos 2x$ ۴

$\cos\left(\frac{\pi}{2}+4x\right) \Rightarrow$  $\rightarrow -\sin 4x = -2\sin 2x\cos 2x$

$\Rightarrow \frac{1}{\cos 2x} - \frac{1}{2\sin 2x\cos 2x} = 0 \rightarrow \frac{1}{\cos 2x} = \frac{1}{2\sin 2x \cos 2x} \rightarrow \sin 2x = \frac{1}{2}$

$\begin{cases} 2x = 2k\pi + \frac{\pi}{6} \rightarrow x = k\pi + \frac{\pi}{12} \rightarrow \frac{\pi}{12} \\ 2x = 2k\pi + \frac{5\pi}{6} \rightarrow x = k\pi + \frac{5\pi}{12} \rightarrow \frac{5\pi}{12} \end{cases} \rightarrow \frac{4\pi}{12} = \frac{\pi}{3} = \alpha$ ۲

$\tan 2\alpha = \tan\frac{2\pi}{3} = -\sqrt{3}$ → ۲

$\cos\left(x+\frac{\pi}{4}\right) = \cos x\cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4} = \boxed{\cos x - \sin x} = \frac{1}{\sqrt{2}}$ ۵

$(\cos x - \sin x)^2 = \underbrace{\cos^2 x + \sin^2 x}_{1} - 2\sin x\cos x = \frac{1}{2}$

$2\sin x\cos x = \boxed{\sin 2x = \cancel{\frac{1}{2}}}$ $\frac{1}{2}$ ۱/۲۵

$m(\cos x - \sin x) - 3\sqrt{6}\sin 2x = \sqrt{6}$

$\hookrightarrow m\left(\frac{1}{\sqrt{2}}\right) - 3\sqrt{6} \times \frac{2}{3} = \sqrt{6} \rightarrow \frac{m}{\sqrt{2}} - 2\sqrt{6} = \sqrt{6}$

$\frac{m}{\sqrt{2}} = 3\sqrt{6} \rightarrow m = 3\sqrt{18} = 9\sqrt{2}$ → ۲

$\sin x\cos\frac{\pi}{6} + \cos x\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x$ ۶

$\cos x\cos\frac{\pi}{3} + \sin x\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x$ ۱/۲۵

$\rightarrow \left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right)^2 = 1 \rightarrow \sin\left(x+\frac{\pi}{6}\right) = \cos\left(x-\frac{\pi}{3}\right) = 1$

$\sin\left(x+\frac{\pi}{6}\right) = 1 \rightarrow x + \frac{\pi}{6} = 2k\pi + \frac{\pi}{2} \rightarrow x = 2k\pi + \frac{\pi}{3}$

$\cos\left(x-\frac{\pi}{3}\right) = 1 \rightarrow x - \frac{\pi}{3} = 2k\pi \rightarrow \boxed{x = 2k\pi + \frac{\pi}{3}}$

$\left\{\frac{\pi}{3}\right\} \rightarrow$ ۴ → ۲

$\sin x + \sqrt{3}\cos x = \sqrt{2} \xrightarrow{\div 2} \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$ ۷

$\to \sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2} \Rightarrow \begin{cases} x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \to x = 2k\pi - \frac{\pi}{12} \\ x + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \to x = 2k\pi + \frac{5\pi}{12} \end{cases}$

$\to x = \left\{-\frac{\pi}{12}, \frac{23\pi}{12}, \frac{5\pi}{12}\right\} \to \frac{23\pi + 5\pi - \pi}{12} = \frac{27\pi}{12} = \frac{9\pi}{4}$ ۴

$\sin\left(\frac{3\pi}{2} - x\right) \Rightarrow$  $\Rightarrow -\cos x$ ۱

$\to -4\sin x\cos x = 1 \to -2\sin 2x = 1 \to \sin 2x = -\frac{1}{2}$

$\begin{cases} 2x = 2k\pi + \frac{7\pi}{6} \to x = k\pi + \frac{7\pi}{12} \\ 2x = 2k\pi + \frac{11\pi}{6} \to x = k\pi + \frac{11\pi}{12} \end{cases} \to \left\{\frac{7\pi}{12}, \frac{19\pi}{12}, \frac{11\pi}{12}, \frac{23\pi}{12}\right\}$ ۲

$\frac{7\pi + 19\pi + 11\pi + 23\pi}{12} = \frac{60\pi}{12} = 5\pi \to ?$

$1 + \cos 2\alpha = 2\cos^2\alpha,\ 1 + \cos 4\alpha = 2\cos^2 2\alpha,\ 1 + \cos 8\alpha = 2\cos^2 4\alpha$ ۴

$\to 8(2\cos^2\alpha) \times (2\cos^2 2\alpha) \times (2\cos^2 4\alpha) = 1$

$\xrightarrow{\times \sin^2\alpha} (\cancel{2\sin^2\alpha\, 2\cos^2\alpha}) \times (\cancel{2\cos^2 2\alpha}) \times (\cancel{2\cos^2 4\alpha}) = \sin^2\alpha$ $\sin^2 4\alpha$ $\sin^2 4\alpha$

$\to \sin^2 8\alpha = \sin^2\alpha \to \sin 8\alpha = \pm\sin\alpha \quad -\sin\alpha = \cancel{\sin(\alpha + \pi)}$

$\begin{cases} 8\alpha = 2k\pi + \alpha \to 7\alpha = 2k\pi \to \alpha = \frac{2k\pi}{7} \\ 8\alpha = 2k\pi + \pi - \alpha \to 9\alpha = 2k\pi + \pi \to \alpha = \frac{(2k+1)\pi}{9} \\ 8\alpha = 2k\pi + \alpha + \pi \to 7\alpha = 2k\pi + \pi \to \alpha = \frac{(2k+1)\pi}{7} \\ 8\alpha = 2k\pi + \pi - \pi - \alpha \to 9\alpha = 2k\pi \to \alpha = \frac{2k\pi}{9} \end{cases}$ ۲

$\max \Rightarrow \cancel{\frac{8\pi}{9}} \to ??$

$\tan ۳x = \frac{۱}{\tan x} = \cot x$ ۱,۷۵ ۱۵

$\tan ۳x = \tan(\frac{\pi}{۲} - x) \longrightarrow ۳x = k\pi \pm \frac{\pi}{۲} - x$

$$\begin{cases} ۳x = k\pi + \frac{\pi}{۲} - x \longrightarrow ۴x = k\pi + \frac{\pi}{۲} \rightarrow x = \frac{k\pi}{۴} + \frac{\pi}{۸} \\ ۳x = k\pi - \frac{\pi}{۲} + x \longrightarrow ۲x = k\pi - \frac{\pi}{۲} \rightarrow x = \frac{k\pi}{۲} - \frac{\pi}{۴} \end{cases}$$

$\{\frac{۹\pi}{۸}, \frac{۱۱\pi}{۸}, \frac{۱۳\pi}{۸}, \frac{۱۵\pi}{۸}, \frac{۵\pi}{۴}, \frac{۷\pi}{۴}\}$

۲۰ ۳۳ ۴۸ ۵۸

$\frac{۹\pi + ۱۱\pi + ۱۳\pi + ۱۵\pi + ۱۰\pi + ۱۴\pi}{۸} = \frac{۷۲\pi}{۸} = ۹\pi$

۵

$$\cos\left(x+\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \rightarrow \cos x - \sin x = \frac{\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \rightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \rightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \rightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}\left(\frac{1}{3}\right) = \sqrt{6}$$

$$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \rightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \rightarrow \boxed{m = 6}$$

۶

$$\cos\left(x - \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3} - x\right)$$

$$\frac{\pi}{3} - x + x + \frac{\pi}{6} = \frac{\pi}{2} \longrightarrow \cos\left(\frac{\pi}{3} - x\right) = \sin\left(x + \frac{\pi}{6}\right)$$

$$\sin^2\left(x + \frac{\pi}{6}\right) = 1 \rightarrow \sin\left(x + \frac{\pi}{6}\right) = \pm 1 \rightarrow x + \frac{\pi}{6} = k\pi + \frac{\pi}{2}$$

$$x = k\pi + \frac{\pi}{3} \xrightarrow{0 \le x \le 2\pi} k=0 \rightarrow x = \frac{\pi}{3}, \quad k=1, \; x = \frac{4\pi}{3}$$

$\boxed{\text{معادله ۲ جواب در بازه دارد}}$

۱۰

$$\tan 3x = \frac{1}{\tan x} = \cot x \rightarrow \tan 3x = \tan\left(\frac{\pi}{2} - x\right)$$

$$3x = k\pi + \frac{\pi}{2} - x \rightarrow 4x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$$

k	4	5	6	7
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$$S = \left(\frac{9+11+13+15}{8}\right)\pi = \frac{48\pi}{8} = \boxed{6\pi}$$