تکلیف ۱۷: بنام خدا یکتا دستانی / دوازدهم دختر ۸

① $8\cos x = 1 + \tan^2 x \rightarrow 8\cos x = \frac{1}{\cos^2 x} \rightarrow 8\cos^3 x = 1$

$\rightarrow \cos x = \frac{1}{2} \rightarrow \begin{cases} 2k\pi + \frac{\pi}{3} \rightarrow \frac{\pi}{3} \\ 2k\pi - \frac{\pi}{3} \rightarrow \frac{5\pi}{3} \end{cases} \rightarrow$ ۲ جواب $\rightarrow$ ۲

② $5 - 5\cos^2 + 2(4\cos^3 - 3\cos) = -2$

$\Rightarrow 5 - 5\cos^2 + 8\cos^3 - 6\cos + 2 = 0 \rightarrow 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

$\cos = -1 \rightarrow$ جواب می‌باشد

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 \;|\; \cos x + 1$

$-8\cos^3 x - 8\cos^2 x \qquad\qquad 8\cos^2 x - 13\cos x + 7$

$-13\cos^2 x - 6\cos x$

$+13\cos^2 x + 13\cos x \qquad\qquad 7\cos x + 7$

$(\cos x + 1)(8\cos^2 x - 13\cos x + 7) = 0 \rightarrow \cos x = -1 \rightarrow$ 

$\cos = -1$ جواب ندارد $2k\pi + \pi \rightarrow \{-\pi, \pi\} \rightarrow$ ۴

③ $2\sin x(\cos^2 x - \sin^2 x) + \sin x = 1$

$2\sin x(1 - 2\sin^2 x) + \sin x = 1 \rightarrow -4\sin^3 + 3\sin = 1$

$\rightarrow \sin 3x = 1 \rightarrow$  $\rightarrow 3x = 2k\pi + \frac{\pi}{2}$

$\rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \rightarrow x = \frac{4k\pi}{6} + \frac{\pi}{6} \rightarrow \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}\right\}$

$\rightarrow \frac{\pi + 5\pi + 9\pi}{6} = \frac{15\pi}{6} = \frac{5\pi}{2} \rightarrow$ ۲

$\sin\left(\frac{\pi}{4}+2x\right) = 0 \rightarrow \cos 2x$ (2)

$\cos\left(\frac{\pi}{4}+2x\right) \Rightarrow \rightarrow -\sin 2x = -2\sin x\cos x$

$\Rightarrow \frac{1}{\cos 2x} - \frac{1}{2\sin x\cos x} = 0 \rightarrow \frac{1}{\cos 2x} = \frac{1}{2\sin x\cos x} \rightarrow \sin 2x = \frac{1}{2}$

$\begin{cases} 2x = 2k\pi + \frac{\pi}{4} \rightarrow x = k\pi + \frac{\pi}{8} \rightarrow \frac{\pi}{8} \\ 2x = 2k\pi + \frac{3\pi}{4} \rightarrow x = k\pi + \frac{3\pi}{8} \rightarrow \frac{3\pi}{8} \end{cases} \rightarrow \frac{2\pi}{8} = \frac{\pi}{4} = \alpha'$

$\tan 2\alpha = \tan\frac{\pi}{2} = \frac{-\sqrt{2}}{2}$?

$\cos\left(x+\frac{\pi}{2}\right) = \cos x\cos\frac{\pi}{2} - \sin x\sin\frac{\pi}{2} = \cos x - \sin x = \frac{1}{\sqrt{2}}$ (3)

$(\cos x - \sin x)^2 = \cos^2 x + \sin^2 x - 2\sin x\cos x = \frac{1}{2}$

$2\sin x\cos x = \sin 2x = \frac{1}{2}$

$m(\cos x - \sin x) - 2\sqrt{2}\sin 2x = \sqrt{2}$

$\hookrightarrow m\left(\frac{1}{\sqrt{2}}\right) - 2\sqrt{2}\times\frac{1}{2} = \sqrt{2} \rightarrow \frac{m}{\sqrt{2}} - \sqrt{2} = \sqrt{2}$

$\frac{m}{\sqrt{2}} = 2\sqrt{2} \rightarrow m = 2\sqrt{18} = 6\sqrt{2}$?

$\sin x\cos\frac{\pi}{4} + \cos x\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}\sin x + \frac{1}{2}\cos x$ (4)

$\cos x\cos\frac{\pi}{4} + \sin x\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}\sin x + \frac{1}{2}\cos x$

$\rightarrow \left(\frac{\sqrt{2}}{2}\sin x + \frac{1}{2}\cos x\right)^2 = 1 \rightarrow \sin\left(x+\frac{\pi}{4}\right)\cdot\cos\left(x-\frac{\pi}{4}\right) = 1$

$\sin\left(x+\frac{\pi}{4}\right) = 1 \rightarrow x+\frac{\pi}{4} = 2k\pi + \frac{\pi}{2} \rightarrow x = 2k\pi + \frac{\pi}{4}$

$\cos\left(x-\frac{\pi}{4}\right) = 1 \rightarrow x - \frac{\pi}{4} = 2k\pi \rightarrow x = 2k\pi + \frac{\pi}{4}$

$2\frac{\pi}{4}\} \rightarrow$ (5) ?

$\sin x + \sqrt{3}\cos x = \sqrt{2} \xrightarrow{\div 2} \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$ (7)

$\to \sin(x + \frac{\pi}{3}) = \frac{\sqrt{2}}{2} \Rightarrow \begin{cases} x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \to x = 2k\pi - \frac{\pi}{12} \\ x + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \to x = 2k\pi + \frac{5\pi}{12} \end{cases}$

$\to x = \{-\frac{\pi}{12}, \frac{23\pi}{12}, \frac{5\pi}{12}\} \to \frac{23\pi + 5\pi - \pi}{12} = \frac{27\pi}{12} = \frac{9\pi}{4}$

4

$\sin(\frac{3\pi}{2} - x) \Rightarrow$  $\Rightarrow -\cos x$ (1)

$\to -4\sin x\cos x = 1 \to -2\sin 2x = 1 \to \sin 2x = -\frac{1}{2}$

$\begin{cases} 2x = 2k\pi + \frac{7\pi}{6} \to x = k\pi + \frac{7\pi}{12} \\ 2x = 2k\pi + \frac{11\pi}{6} \to x = k\pi + \frac{11\pi}{12} \end{cases} \to \{\frac{7\pi}{12}, \frac{19\pi}{12}, \frac{11\pi}{12}, \frac{23\pi}{12}\}$

$\frac{7\pi + 19\pi + 11\pi + 23\pi}{12} = \frac{60\pi}{12} = 5\pi \to 2.$

$1 + \cos 2\alpha = 2\cos^2\alpha \,,\; 1 + \cos 4\alpha = 2\cos^2 2\alpha \,,\; 1 + \cos 8\alpha = 2\cos^2 4\alpha$ (9)

$\to 8(2\cos^2\alpha) \times (2\cos^2 2\alpha) \times (2\cos^2 4\alpha) = 1$

$\xrightarrow{\times \sin^2\alpha} (2\sin^2\alpha\, 2\cos^2\alpha) \times (2\cos^2 2\alpha) \times (2\cos^2 4\alpha) = \sin^2\alpha$ ($\sin^2 4\alpha$, $\sin^2 8\alpha$)

$\to \sin^2 8\alpha = \sin^2\alpha \to \sin 8\alpha = +\sin\alpha$, $-\sin\alpha$ ~~$\sin(\alpha + \pi)$~~

$\begin{cases} 8\alpha = 2k\pi + \alpha \to 7\alpha = 2k\pi \to \alpha = \frac{2k\pi}{7} \\ 8\alpha = 2k\pi + \pi - \alpha \to 9\alpha = 2k\pi + \pi \to \alpha = \frac{(2k+1)\pi}{9} \\ 8\alpha = 2k\pi + \alpha + \pi \to 7\alpha = 2k\pi + \pi \to \alpha = \frac{(2k+1)\pi}{7} \\ 8\alpha = 2k\pi + \pi - \pi - \alpha \to 9\alpha = 2k\pi \to \alpha = \frac{2k\pi}{9} \end{cases}$

$\max \Rightarrow \frac{8\pi}{9} \to 2?$

Page : ( )

SUBJECT :

Year: Month: Day:

$\tan 2\alpha = \frac{1}{\tan \alpha} = \cot \alpha$ (۱۵)

$\tan 2\alpha = \tan\left(\frac{\pi}{2} - \alpha\right) \longrightarrow 2\alpha = k\pi + \frac{\pi}{2} - \alpha$

$\begin{cases} 2\alpha = k\pi + \frac{\pi}{2} - \alpha \longrightarrow 4\alpha = k\pi + \frac{\pi}{2} \rightarrow \alpha = \frac{k\pi}{4} + \frac{\pi}{8} \\ 2\alpha = k\pi - \frac{\pi}{2} + \alpha \longrightarrow 2\alpha = k\pi - \frac{\pi}{2} \rightarrow \alpha = \frac{k\pi}{2} - \frac{\pi}{4} \end{cases}$

$\left\{\frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

$\frac{9\pi + 11\pi + 13\pi + 15\pi + 10\pi + 14\pi}{8} = \frac{77\pi}{8} = 9\pi$