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دوازدهم A — تکلیف ۱۷ — سارینا رضوی

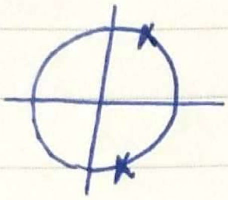
۱) $[0, 2\pi]$

$8\cos x - \tan^2 x = 1 \rightarrow 1 + \tan^2 x = 8\cos x$

$\frac{1}{\cos^2 x} = 8\cos x \xrightarrow{\times \cos^2 x} 8\cos^3 x = 1$

$\cos^3 x = \frac{1}{8}$

$\cos x = \frac{1}{2}$



→ ۲ جواب ✓

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۲) $[-\pi, \pi]$

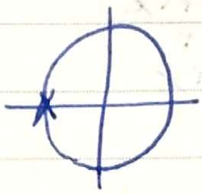
$5\sin^2 x + 2\cos 3x = -2$

$\sin^2 x = 1 - \cos^2 x$

$\cos 3x = 4\cos^3 x - 3\cos x$

طبق دامنه ۲ جواب داریم

π و $-\pi$ ✓



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$\rightarrow 5 - 5\cos^2 x + 8\cos^3 x - 6\cos x + 2 = 0 \rightarrow 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

$\rightarrow 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0 \rightarrow$ جمع ضرایب توان های زوج با جمع ضرایب توان های فرد برابر است. یکی از ریشه ها (-1) است.

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 \;|\; \cos x + 1 \rightarrow$ فقط به ازای (-1)

$8\cos^2 x - 13\cos x + 7 \rightarrow$ همواره (+) ریشه ندارد

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۳)

$2\sin x \cos 2x + \sin x = 1$

$2\sin x(\cos^2 x - \sin^2 x) + \sin x = 1 \rightarrow \sin x(2\cos^2 x - 2\sin^2 x + 1) = 1$

$2\cos^2 x = 2 - 2\sin^2 x$ ، $3 - 4\sin^2 x$

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$-4\sin^3 x + 3\sin x - 1 = 0 \rightarrow \sin 3x = 1$

($-4\sin^3 x + 3\sin x = \sin 3x$)

$3x = 2k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6}$

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$x = \frac{2k\pi}{2} + \frac{\pi}{4}$

- $k=0 \rightarrow \frac{\pi}{4}$ ✓
- $k=1 \rightarrow \frac{5\pi}{4}$ ✓
- $k=2 \rightarrow \frac{9\pi}{2}$ ✓
- $k=3 \rightarrow \frac{13\pi}{4}$ ✗

$\frac{3\pi}{2} + \frac{5\pi}{4} + \frac{\pi}{4} = \frac{10\pi}{4}$ ✓

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(۴)

$$\frac{1}{\sin\left(\frac{\pi+4x}{2}\right)} + \frac{1}{\cos\left(\frac{\pi+8x}{2}\right)}$$

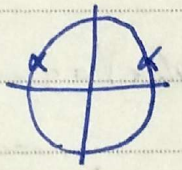
$\sin\left(\frac{\pi+4x}{2}\right) = \cos 2x$

$\cos\left(\frac{\pi+8x}{2}\right) = -\sin 4x = -2\sin 2x\cos 2x$

$$= \frac{1}{\cos 2x} + \frac{1}{-2\sin 2x\cos 2x} = 0 \rightarrow \frac{1}{\cos 2x}\left(1 + \frac{1}{-2\sin 2x}\right) = 0$$ (۲)

$-2\sin 2x = -1$

$\sin 2x = \frac{1}{2}$



$2x = \frac{\pi}{6} \rightarrow x = \frac{\pi}{12}$

$2x = \frac{5\pi}{6} \rightarrow x = \frac{5\pi}{12}$

$\alpha = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{4\pi}{12} \rightarrow \frac{\pi}{3} \rightarrow \tan^2\alpha = \tan^2\frac{\pi}{3} = \boxed{-\sqrt{3}}$

$m(\cos x - \sin x) - 2\sqrt{2}\sin(2x) = \sqrt{x+4}$ (۵)

$\cos\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ (۰)

$\cos\left(x+\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{2}}{2} \rightarrow \cos x - \sin x = \frac{1}{\frac{\sqrt{2}}{2}}$

$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{2}} \xrightarrow{\text{به توان ۲}} \frac{\sqrt{4}}{2} \rightarrow \boxed{\cos x - \sin x = \frac{\sqrt{4}}{2}}$

$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{4}{9} \rightarrow \boxed{\sin 2x = \frac{1}{2}}$

$m(\cos x - \sin x) - 2\sqrt{4}(\sin 2x) = \sqrt{4} \rightarrow \frac{\sqrt{4}}{2}m - 2\sqrt{4}\left(\frac{1}{2}\right) = \sqrt{4}$

$\frac{\sqrt{4}}{2}m - \sqrt{4} = \sqrt{4} \rightarrow \frac{m\sqrt{4}}{2} = 2\sqrt{4} \rightarrow \boxed{m = 4}$

Subject:

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۴) $\sin\left(x+\frac{\pi}{4}\right)\cos\left(x-\frac{\pi}{3}\right)=1 \rightarrow 2\cos\left(x-\frac{\pi}{3}\right)=1$ (۱٫۵)

$\cos\left(x-\frac{\pi}{3}\right)$

$\cos\left(x-\frac{\pi}{3}\right)=\frac{1}{2}$

$x-\frac{\pi}{3}\begin{cases}\rightarrow 2k\pi+\frac{\pi}{3}\\ \rightarrow 2k\pi-\frac{\pi}{3}\end{cases} \rightarrow x=\left\{2k\pi+\frac{2\pi}{3}, 2k\pi\right\}$

در بازه $[0,2\pi]$ ← ۳ جواب داره

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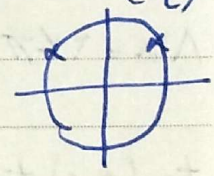
۷) $[-\pi,2\pi]$

$\sin x+\sqrt{3}\cos x=\sqrt{2}$

$\times\frac{1}{2} \rightarrow \left(\sin x\times\frac{1}{2}\right)+\left(\frac{\sqrt{3}}{2}\times\cos x\right)=\frac{\sqrt{2}}{2}$

$\sin x\cos\frac{\pi}{3}+\cos x\sin\frac{\pi}{3}=\sin\frac{\pi}{4} \rightarrow \sin\left(x+\frac{\pi}{3}\right)=\sin\left(\frac{\pi}{4}\right)$



$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \rightarrow x=\frac{23\pi}{12}, -\frac{\pi}{12}$

$x+\frac{\pi}{3}=2k\pi+2\frac{\pi}{4} \rightarrow \frac{5\pi}{12}$

$\frac{23\pi}{12}+\frac{5\pi}{12}-\frac{\pi}{12}=\boxed{\frac{9\pi}{4}}$

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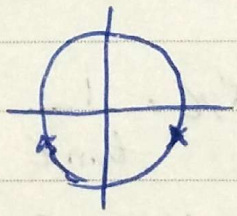
۸) $4\sin x\sin\left(\frac{3\pi}{2}-x\right)=1 \rightarrow -4\sin x\cos x=1$ (۱٫۵)

$-\cos x$

$-2\sin 2x=1 \rightarrow \sin 2x=-\frac{1}{2}$



$2x\begin{cases}\rightarrow 2k\pi-\frac{\pi}{4}\\ \rightarrow 2k\pi-\frac{5\pi}{4}\end{cases} \rightarrow x\begin{cases}\rightarrow k\pi-\frac{\pi}{12}\\ \rightarrow k\pi-\frac{5\pi}{12}\end{cases}$

۴ جواب ←

مضبوع جواب ها !

$$\left(1+\cos(۲\alpha)\right)\left(1+\cos(۴\alpha)\right)\left(1+\cos(۸\alpha)\right)=\frac{1}{۸}$$ ۱۹

$۲\cos^۲\alpha \quad ۲\cos^۲ ۲\alpha \quad ۲\cos^۲ ۴\alpha$

$$\xrightarrow{\times}\ \frac{\sin^۲\alpha}{\sin^۲\alpha}\to\frac{۸\left(\sin^۲\alpha\cos^۲\alpha\cos^۲ ۲\alpha\cos^۲ ۴\alpha\right)}{\sin^۲\alpha}=\frac{1}{۸}$$

$\frac{1}{۴}\sin^۲ ۲\alpha \to \frac{1}{۶۴}\sin^۲ ۸\alpha$

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$$\to\frac{1}{۶۴}\sin^۲ ۸\alpha\times\frac{۸}{\sin^۲\alpha}=\frac{1}{۸}\to\sin^۲ ۸\alpha=\sin^۲\alpha$$

$\to \sin ۸\alpha=\sin\alpha$

$\to \sin ۸\alpha=\sin(-\alpha)=-\sin\alpha$

$۸\alpha=۲k\pi+\alpha\to\alpha=\frac{۲k\pi}{۷}$

$۸\alpha=۲k\pi+\pi-\alpha\to\alpha=\frac{۲k\pi+\pi}{۹}$

$\to \max_\alpha=\frac{۸\pi}{۹}$

$۸\alpha=۲k\pi-\alpha\to\alpha=\frac{۲k\pi}{۹}$

$۸\alpha=۲k\pi+\pi+\alpha\to\alpha=\frac{۲k\pi+\pi}{۷}$

$\tan(۳x)\tan(x)=1$ $[\pi, ۲\pi]$ ۱۰

[redacted]

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$$\tan ۳x=\frac{1}{\tan x}\to\tan ۳x=\cot x\to\tan ۳x=\tan\left(\frac{\pi}{۲}-x\right)$$

$$۳x=k\pi+\frac{\pi}{۲}-x\to x=\frac{k\pi}{۴}+\frac{\pi}{۸}\to[\pi, ۲\pi]$$

$$x\to\frac{۹\pi}{۸}+\frac{۱۱\pi}{۸}+\frac{۱۳\pi}{۸}+\frac{۱۵\pi}{۸}\to ۶\pi$$

(۶)

$\cos(x-\frac{\pi}{2}) = \cos(\frac{\pi}{2}-x)$

$\frac{\pi}{2}-x+x+\frac{\pi}{2} = \pi$ ⟶ $\cos(\frac{\pi}{2}-x) = \sin(x+\frac{\pi}{2})$

$\sin^2(x+\frac{\pi}{4}) = 1 \rightarrow \sin(x+\frac{\pi}{4}) = \pm 1 \rightarrow x+\frac{\pi}{4} = k\pi+\frac{\pi}{2}$

$x = k\pi+\frac{\pi}{4} \xrightarrow{0\le x<2\pi}$ $k=0 \rightarrow x=\frac{\pi}{4}$ ، $k=1, x=\frac{5\pi}{4}$

معادله ۲ جواب دربازه دارد

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(۷)

$\sin(\frac{3\pi}{2}-x) = -\cos x$

$4\sin x(-\cos x) = 1 \rightarrow -2(2\sin x\cos x) = 1 \rightarrow \sin 2x = -\frac{1}{2}$

$2x = 2k\pi-\frac{\pi}{6}$ (2) یا $2k\pi+\frac{7\pi}{6}$ (1) $\rightarrow x = k\pi-\frac{\pi}{12}$ (2) یا $k\pi+\frac{7\pi}{12}$ (1)

$0\le x<2\pi$ ⟶ (1) $k=0,1 \rightarrow x=\frac{7\pi}{12}, \frac{19\pi}{12}$

(2) $k=1,2 \rightarrow x=\frac{11\pi}{12}$ و $\frac{23\pi}{12}$

$S = \frac{7\pi}{12}+\frac{9\pi}{12}+\frac{11\pi}{12}+\frac{23\pi}{12} = \frac{50\pi}{12} = 4\pi$