پریا شهری‌نیا

۱- $[0, 2\pi]$

$$8\cos x - \tan^2 x = 1 \xrightarrow{1+\tan^2 x = \frac{1}{\cos^2 x} \to \tan^2 x = \frac{1}{\cos^2 x} - 1} 8\cos x - \left(\frac{1}{\cos^2 x} - 1\right) = 1 \longrightarrow$$

$$\to 8\cos x - \frac{1}{\cos^2 x} = 0 \longrightarrow 8\cos^3 x - 1 = 0 \to \cos^3 x = \frac{1}{8} \to \cos x = \frac{1}{2}$$

$x = \frac{\pi}{3}, \frac{5\pi}{3} \longrightarrow$ ۲ ریشه دارد

۲- $[-\pi, \pi]$

$$5\sin^2(x) + 2\cos(3x) = -2$$

$$5\sin^2 x = 5(1-\cos^2 x), \quad 2\cos 3x = 2(4\cos^3 x - 3\cos x) = 8\cos^3 x - 6\cos x$$

$$5 - 5\cos^2 x + 8\cos^3 x - 6\cos x = -2 \to 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0 \xrightarrow{\text{حدس ریشه}} \cos x = -1 \to$$

$$\to (\cos x + 1)\underbrace{(8\cos^2 x - 13\cos x + 7)}_{\text{ریشه ندارد } \Delta < 0} \to \cos x = -1 \to x = \pi, -\pi \to$$ ۲ جواب دارد

۳-

$$2\sin(x)\cos(2x) + \sin(x) = 1$$

$$2\sin x(1 - 2\sin^2 x) + \sin x = 1 \to 2\sin x - 4\sin^3 x + \sin x = 1 \to \underbrace{3\sin x - 4\sin^3 x}_{\sin 3x} = 1 \to$$

$$\to \sin 3x = 1 \to 3x = 2k\pi + \frac{\pi}{2} \to x = \frac{2k\pi}{3} + \frac{\pi}{6}$$

$k=0 \to \frac{\pi}{6}$

$k=1 \to \frac{2\pi}{3} + \frac{\pi}{6} = \frac{5\pi}{6} \quad \to \frac{\pi}{6} + \frac{5\pi}{6} + \frac{3\pi}{2} = \frac{5\pi}{2}$

$k=2 \to \frac{4\pi}{3} + \frac{\pi}{6} = \frac{3\pi}{2}$

۴-

$$\frac{1}{\sin\left(\frac{\pi + 4x}{2}\right)} + \frac{1}{\cos\left(\frac{\pi + 8x}{2}\right)} = 0$$

$\sin(\frac{\pi}{2} + 2x) = \cos 2x$

$\cos(\frac{\pi}{2} + 4x) = -\sin 4x$

$$\to \frac{1}{\cos 2x} - \frac{1}{\sin 4x} = 0 \to$$

$$\to \frac{1}{\cos 2x} = \frac{1}{\sin 4x} \longrightarrow \sin 4x = \cos 2x \to 2\sin 2x\cos 2x = \cos 2x \to 2\sin 2x\cos 2x - \cos 2x = 0 \to$$

$\to \cos 2x(2\sin 2x - 1) = 0$

$\cos 2x = 0 \to$ غ ق ق $\leftarrow$ چون مخرج را صفر می‌کند

$\sin 2x = \frac{1}{2}$: $2x = \frac{\pi}{6} \to x = \frac{\pi}{12}$; $2x = \frac{5\pi}{6} \to x = \frac{5\pi}{12}$

$$\to \alpha = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$$

$$\tan 2\left(\frac{\pi}{3}\right) = \tan\frac{2\pi}{3} = -\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$$

۵ـ

$m(\cos x - \sin x) - 3\sqrt{6}\sin(2x) = \sqrt{6}$, if $\cos\left(x+\frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$ $m = ?$

$\cos\left(x+\frac{\pi}{4}\right) = \frac{1}{\sqrt{3}} \to \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}} \to \cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}}$

$(\cos x - \sin x)^2 = \frac{2}{3} \to 1 - 2\sin x\cos x = \frac{2}{3} \to 1 - \sin 2x = \frac{2}{3} \to \sin 2x = \frac{1}{3}$

$\sqrt{\frac{2}{3}}\, m - \frac{3\sqrt{6}}{3} = \sqrt{6} \to \sqrt{\frac{2}{3}}\, m = 2\sqrt{6} \to \frac{\sqrt{2}}{\sqrt{3}}\, m = 2\sqrt{6} \to m = 6$

۶ـ

$\sin\left(x+\frac{\pi}{6}\right)\underbrace{\cos\left(x-\frac{\pi}{3}\right)}_{\cos\left(\frac{\pi}{3}-x\right)} = 1 \longrightarrow x+\frac{\pi}{6}+\frac{\pi}{3}-x = \frac{\pi}{2} \to$ دو کمان متمم هستند ← $\sin\alpha = \cos\beta$

$\to \sin\left(x+\frac{\pi}{6}\right) = \cos\left(x-\frac{\pi}{3}\right) \to \cos^2\left(x-\frac{\pi}{3}\right) = 1 \to \cos\left(x-\frac{\pi}{3}\right) = \pm 1$

$\to \cos\left(x-\frac{\pi}{3}\right) = 1 \to x-\frac{\pi}{3} = 2k\pi \to x = 2k\pi + \frac{\pi}{3} \xrightarrow{k=0} \frac{\pi}{3}$

$\to \cos\left(x-\frac{\pi}{3}\right) = -1 \to x-\frac{\pi}{3} = 2k\pi+\pi \to x = 2k\pi + \frac{4\pi}{3} \xrightarrow{k=0} \frac{4\pi}{3}$

۲ جواب دارد ←

۷ـ

$\sin x + \sqrt{3}\cos x = \sqrt{2} \xrightarrow{\div 2} \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2} \to \cos\frac{\pi}{3}\sin x + \sin\frac{\pi}{3}\cos x = \frac{\sqrt{2}}{2} \to$

$\to \sin\left(x+\frac{\pi}{3}\right) = \sin\frac{\pi}{4}$

$\to x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \to x = 2k\pi - \frac{\pi}{12} \xrightarrow{k=0} -\frac{\pi}{12}$; $\xrightarrow{k=1} \frac{23\pi}{12}$

$\to x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \to x = 2k\pi + \frac{5\pi}{12} \xrightarrow{k=0} \frac{5\pi}{12}$

$-\frac{\pi}{12} + \frac{23\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \frac{9\pi}{4}$ ← جواب

۸ـ

$4\sin x \underbrace{\sin\left(\frac{3\pi}{2}-x\right)}_{-\cos x} = 1 \to -4\sin x\cos x = 1 \to \sin x\cos x = -\frac{1}{4} \to \frac{1}{2}\sin 2x = -\frac{1}{4} \to$

$\to \sin 2x = -\frac{1}{2}$

$\to 2x = 2k\pi - \frac{\pi}{6} \to x = k\pi - \frac{\pi}{12} \xrightarrow{k=1} \frac{11\pi}{12}$; $\xrightarrow{k=2} \frac{23\pi}{12}$

$\to 2x = 2k\pi + \frac{7\pi}{6} \to x = k\pi + \frac{7\pi}{12} \xrightarrow{k=0} \frac{7\pi}{12}$; $\xrightarrow{k=1} \frac{19\pi}{12}$

$\frac{11\pi}{12} + \frac{23\pi}{12} + \frac{7\pi}{12} + \frac{19\pi}{12} = 5\pi$

9-

$$\underbrace{(1+\cos(2\alpha))}_{2\cos^2\alpha}\underbrace{(1+\cos(4\alpha))}_{2\cos^2 2\alpha}\underbrace{(1+\cos(8\alpha))}_{2\cos^2 4\alpha}=\frac{1}{8}$$

$$8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha=\frac{1}{8}\to\cos\alpha\cos2\alpha\cos4\alpha=\pm\frac{1}{8}\xrightarrow[\text{طرفین ضرب در } 8\sin\alpha]{\sin\alpha\neq0}\underbrace{\overbrace{\overbrace{8\sin\alpha\cos\alpha}^{4\sin2\alpha}\cos2\alpha}^{2\sin4\alpha}\cos4\alpha}_{\sin8\alpha}=\pm\sin\alpha$$

$$\to\sin8\alpha=\pm\sin\alpha$$

$\to \alpha=\frac{2k\pi}{7}\xrightarrow{k=3}\frac{6\pi}{7}$

$\to \alpha=\frac{2k\pi}{9}\xrightarrow{k=4}\frac{8\pi}{9}$

$\to \alpha=\frac{2k\pi}{7}+\frac{\pi}{7}\xrightarrow{k=2}\frac{4\pi}{7}$

$\to \alpha=\frac{2k\pi}{9}+\frac{\pi}{9}\xrightarrow{k=3}\frac{7\pi}{9}$

$\to \max=\frac{8\pi}{9}$ جواب

$k=0$ را بخاطر قرار ندادیم که جواب برابر با π نشود چون $\sin\alpha\neq0$ است

10-

$$\tan(3x)\tan(x)=1\to\tan3x=\frac{1}{\tan x}\to\tan3x=\cot x\to\tan3x=\tan\left(\frac{\pi}{2}-x\right)\to$$

$$\to3x=k\pi+\frac{\pi}{2}-x\to x=\frac{k\pi}{4}+\frac{\pi}{8}$$

$\xrightarrow{k=4}\frac{9\pi}{8}$

$\xrightarrow{k=5}\frac{11\pi}{8}$

$\xrightarrow{k=6}\frac{13\pi}{8}$

$\xrightarrow{k=7}\frac{15\pi}{8}$

$$\frac{9\pi}{8}+\frac{11\pi}{8}+\frac{13\pi}{8}+\frac{15\pi}{8}=6\pi$$ جواب