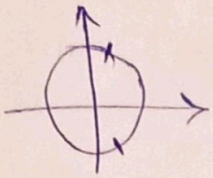
۱۹/۵ آخرین به سمت

نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

۱

$\wedge\cos x = 1+\tan^2 x \Rightarrow \wedge\cos x = \frac{1}{\cos^2 x} \Rightarrow \wedge\cos^3 x = 1 \Rightarrow \cos^3 x = \frac{1}{\wedge}$

$\Rightarrow \cos x = \frac{1}{2}$



در بازه $[0, 2\pi]$ دارای ۲ جواب $(\frac{\pi}{3}$ و $\frac{5\pi}{3})$ است.

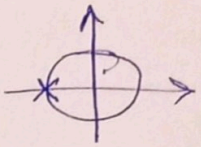
۲

$5\sin^2 x + 2\cos^3 x = -2 \Rightarrow 5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) = -2$

$\wedge\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0 \quad (a+c=b+d) \Rightarrow$

$(\cos x + 1)(\wedge\cos^2 x - 13\cos x + 7) = 0 \Rightarrow \cos x = -1$

$\Delta < 0$



در بازه $[-\pi, 3\pi]$ دارای ۲ جواب است.

۳

$2\sin x \cos 2x + \sin x = 1 \Rightarrow 2\sin x(1-2\sin^2 x) + \sin x = 1$

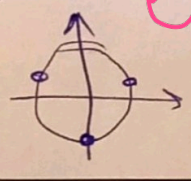
$\Rightarrow 4\sin^3 x - 2\sin x + 1 = 0 \quad (a+b=c) \Rightarrow (\sin x + 1)(4\sin^2 x - 4\sin x + 1) = 0$

$(2\sin x - 1)^2$

$\sin x = -1 \quad \frac{3\pi}{2} = x$

$\sin x = \frac{1}{2} \quad x = \frac{\pi}{6} \to \frac{5\pi}{6}$

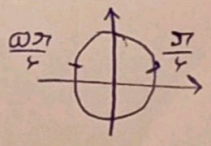
$\frac{\pi}{6} + \frac{5\pi}{6} + \frac{3\pi}{2} = \frac{5\pi}{2}$



۴

$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = \frac{1}{\cos 2x} + \frac{1}{-2\sin 2x\cos 2x} =$

$\frac{-2\sin 2x + 1}{-2\sin 2x\cos 2x} = 0 \Rightarrow -2\sin 2x + 1 = 0 \Rightarrow \sin 2x = \frac{1}{2}$



$2x = \frac{\pi}{6} \quad x = \frac{\pi}{12}$

$2x = \frac{5\pi}{6} \quad x = \frac{5\pi}{12}$

اختلاف $= \frac{5\pi}{12} - \frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3} \quad \tan\frac{2\pi}{3} = \tan 120^\circ = -\sqrt{3}$

۵

$m(\cos x - \sin x) - 3\sqrt{6}\sin(2x) = \sqrt{6} \qquad \sin x + \cos x = \sqrt{2}\sin(x + \frac{\pi}{4})$

$-m\sqrt{2}\sin(x - \frac{\pi}{4}) - 3\sqrt{6}\sin(2x) = \sqrt{6} \Rightarrow -m\sin(x - \frac{\pi}{4}) - 3\sqrt{3}\sin(2x) = \sqrt{3}$

$\frac{\sqrt{3}}{3}m - 3\sqrt{3}\sin 2x = \sqrt{3} \Rightarrow m - 9\sin 2x = 3 \quad m = 6$

$\frac{1}{3}$

$\cos(x + \frac{\pi}{4}) = \frac{\sqrt{3}}{3} \Rightarrow \sin(\underbrace{x + \frac{\pi}{4}}_{\alpha}) = \frac{\sqrt{6}}{3}$

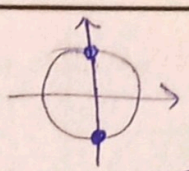
$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2 \times \frac{\sqrt{6}}{3} \times \frac{\sqrt{3}}{3} = \frac{2\sqrt{2}}{3}$

$\sin(2x + \frac{\pi}{2}) = \frac{2\sqrt{2}}{3} \quad \cos 2x = \frac{2\sqrt{2}}{3} \to \sin 2x = \frac{1}{3}$ (طبق گزینه‌ها)

۶

$\sin\left(x+\frac{\pi}{4}\right)\cos\left(x-\frac{\pi}{3}\right)=1 \quad (\cos(\alpha)=\cos(-\alpha))$

$\sin\left(\underbrace{x+\frac{\pi}{4}}_{\alpha}\right)\cos\left(\underbrace{\frac{\pi}{3}-x}_{\beta}\right)=1 \quad \alpha+\beta=90^\circ$



$\sin\left(x+\frac{\pi}{4}\right)\sin\left(x+\frac{\pi}{4}\right)=1 \Rightarrow \sin^2\left(x+\frac{\pi}{4}\right)=1$

$\sin\left(x+\frac{\pi}{4}\right)=1 \quad x+\frac{\pi}{4}=\frac{\pi}{2} \quad x=\frac{\pi}{4}$

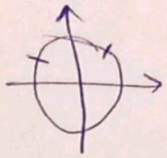
$\sin\left(x+\frac{\pi}{4}\right)=-1 \quad x+\frac{\pi}{4}=\frac{3\pi}{2} \quad x=\frac{5\pi}{4}$

دارای ۲ جواب است ✓

۷

$\sin x+\sqrt{3}\cos x=\sqrt{2} \quad \left(a\sin x+b\cos x=\frac{a}{|a|}\sqrt{a^2+b^2}\sin(x+\alpha)\right)$

$\Rightarrow 2\sin\left(x+\frac{\pi}{3}\right)=\sqrt{2} \Rightarrow \sin\left(x+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2}$



$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \Rightarrow x=2k\pi-\frac{\pi}{12}$

$x+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4} \Rightarrow x=2k\pi+\frac{5\pi}{12}$

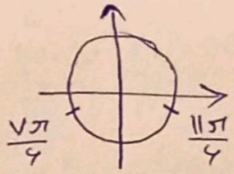
$x=-\frac{\pi}{12}, \frac{23\pi}{12}, \frac{5\pi}{12}$ ✓

مجموع: $2\pi+\frac{5\pi}{12}-\frac{\pi}{12}=2\pi+\frac{\pi}{3}=2\pi+\frac{\pi}{3}=\frac{7\pi}{3}$

۸

$4\sin x\sin\left(\frac{3\pi}{2}-x\right)=1 \Rightarrow 4\sin x(-\cos(x))=1 \Rightarrow -4\sin x\cos x=1$

$(2\sin x\cos x=\sin 2x) \Rightarrow -2\sin 2x=1 \Rightarrow \sin 2x=-\frac{1}{2}$



$2x=2k\pi-\frac{\pi}{6} \quad x=k\pi-\frac{\pi}{12}$

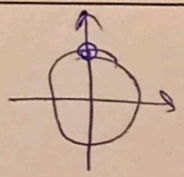
$2x=2k\pi-\frac{5\pi}{6} \quad x=k\pi-\frac{5\pi}{12}$

جواب‌ها $=\pi-\frac{\pi}{12}, 2\pi-\frac{\pi}{12}, \pi-\frac{5\pi}{12}, 2\pi-\frac{5\pi}{12}$ ✓ مجموع $=4\pi-\pi=3\pi$

۹

$1+\cos 2\alpha=2\cos^2\alpha \quad 1+\cos 4\alpha=2\cos^2 2\alpha \quad 1+\cos 8\alpha=2\cos^2 4\alpha$

$(2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)=\frac{1}{8} \Rightarrow (\cos^2\alpha)(\cos^2 2\alpha)(\cos^2 4\alpha)=\frac{1}{64}$



$\Rightarrow (\cos\alpha)(\cos 2\alpha)(\cos 4\alpha)=\pm\frac{1}{8} \times \frac{\sin\alpha}{\sin\alpha}$

$\frac{1}{2}\sin 2\alpha\cos 2\alpha\cos 4\alpha=\pm\frac{1}{8}$ (۱/۵)

$\frac{1}{4}\sin 4\alpha$

$\Rightarrow \frac{1}{8}\sin 8\alpha=\frac{1}{8} \Rightarrow \sin 8\alpha=1$

طرفین عبارت در $\sin\alpha$ ضرب می‌شود!

$\sin 8\alpha=\pm\sin\alpha$

$8\alpha=2k\pi+\frac{\pi}{2} \quad \alpha=\frac{k\pi}{4}+\frac{\pi}{16} \quad \max_{k=2}=\frac{13\pi}{16}$

۱۰

$\tan(3x)=\frac{1}{\tan x} \Rightarrow \tan(3x)=\cot x \Rightarrow \tan(3x)=\tan\left(\frac{\pi}{2}-x\right)$

$\Rightarrow 3x=k\pi+\frac{\pi}{2}-x \Rightarrow x=\frac{k\pi}{4}+\frac{\pi}{8}$

جواب‌ها $=$ $\frac{9\pi}{8}+\frac{11\pi}{8}+\frac{13\pi}{8}+\frac{15\pi}{8}=\frac{48\pi}{8}=6\pi$

$k=4,5,6,7$ ✓

(۹)

$$1+\cos 2\star = 2\cos^2\star$$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \xrightarrow{?} 8\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha \cos 2\alpha \cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha \ \star}$$

$$\underbrace{(\sin\alpha \cos\alpha)}_{\frac{1}{2}\sin 2\alpha} \cos 2\alpha \cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha \cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha} \cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha \cos 4\alpha)}_{\sin 8\alpha} = \frac{\pm\sin\alpha}{8} \rightarrow \frac{1}{8}\sin 8\alpha = \pm\frac{\sin\alpha}{8}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا} \ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا} \ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \rightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \rightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \rightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \rightarrow x = \frac{7\pi}{9}$$

$$\rightarrow \boxed{\max A = \frac{8\pi}{9}}$$

A ★ نمی‌تواند برابر خود π باشد چون طبق ★ $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!