۱۸/۵

نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره .... کلاس ....

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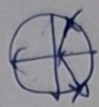
$8\cos x = \frac{1}{\cos^2 x} \to 8\cos^3 x = 1 \to \cos x = \frac{1}{2}$



در $[0, 2\pi]$ ← ۲ جواب

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$5s^2x \to 5 - 5c^2x$

$2\cos 3x = 2(4c^3 - 3c) = 8c^3 - 6c$ } $\to 8c^3 - 5c^2 - 6c + 7 = 0$

$c = -1$

$(c+1)(8c^2 - 13c + 7)$

$\Delta = 169 - 224 < 0$

جواب دیگری ندارد.

دوباره $[0, 2\pi]$ ← دو جواب دارد.

$\to \pi$ و $-\pi$

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$\sin x(2\cos(2x) + 1) = 1 \to \sin x(3 - 4s^2) = 1 \to 4s^3 - 3s + 1 = 0$

$2(1 - 2s^2(x))$

$(s+1)(4s^2 - 4s + 1)$

$(2s - 1)^2$

$s = -1$ ، $\frac{1}{2}$

$\frac{5\pi}{6}, \frac{\pi}{6} \leftarrow$ در $[0, 2\pi]$

$\frac{3\pi}{2} \to \frac{5\pi}{2}$ مجموع

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$\frac{1}{\sin(\frac{\pi}{2}+3x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = \frac{1}{\cos(3x)} - \frac{1}{\sin(4x)} = 0 \to \sin(4x) - \cos 3x = 0$

$\sin 4x = \cos 3x$ ← در لحظه‌ای باید $\sin 4x$ و $\cos 3x$ صفر نشوند

$\div \cos 3x$

$2\sin 3x = 1 \to \sin 3x = \frac{1}{2} \to x = \frac{5\pi}{12}$ ، $x = \frac{\pi}{12}$

اضافه $\alpha = \frac{\pi}{3}$

$\tan 3\alpha = -\sqrt{3}$

$\frac{3\pi}{2}$

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$\cos(x+\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}} \to \cos x - \sin x = \frac{2}{\sqrt{6}} = \frac{\sqrt{6}}{3}$

توان ۲

$c^2 + s^2 - \sin 2x = \frac{2}{3}$

$\sin 2\alpha = \frac{1}{3}$

$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \to m = 6$

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$\frac{\pi}{3}-x=\frac{\pi}{2}-(x+\frac{\pi}{6})$, $\cos(x-\frac{\pi}{3})=\cos(\frac{\pi}{3}-x)$ $s(x+\frac{\pi}{6})^2 = 1$ $\begin{cases} s(x+\frac{\pi}{6})=1 \xrightarrow{\text{[illegible]}} x+\frac{\pi}{6}=2k\pi \rightarrow x=2k\pi-\frac{\pi}{6} \quad \frac{11\pi}{6} \\ s(x+\frac{\pi}{6})=-1 \rightarrow x+\frac{\pi}{6}=(2k+1)\pi \rightarrow x=2k\pi+\frac{5\pi}{6} \quad \frac{5\pi}{6} \end{cases}$ ۲ جواب ۲	۶	
توان ۲ $\rightarrow s^2+3c^2+2\sqrt{3}sc=2 \rightarrow 2c^2+2\sqrt{3}sc=1 \rightarrow c^2+2\sqrt{3}cs=s^2$ $-\sqrt{3}s_{2x}=\cos_{2x} \rightarrow \cot_{2x}=-\sqrt{3} \rightarrow 2x=k\pi-\frac{\pi}{6} \rightarrow x=\frac{k\pi}{2}-\frac{\pi}{12}$ $k=2=\pi-\frac{\pi}{12}$, $k=-1 \rightarrow -\frac{\pi}{2}-\frac{\pi}{12}$ 4π : مجموع $\leftarrow$ $k=3=\frac{3\pi}{2}-\frac{\pi}{12}$, $k=0=-\frac{\pi}{12}$ $k=4=2\pi-\frac{\pi}{12}$, $k=1=\frac{\pi}{2}-\frac{\pi}{12}$ ۱/۷۵	۷	
$\sin(\frac{3\pi}{2}-x)=-\cos x$, $-4sc=1 \rightarrow -2s_{2x}=1 \rightarrow \sin_{2x}=-\frac{1}{2}$ $2x \rightarrow 2k\pi-\frac{\pi}{6} \rightarrow x=k\pi-\frac{\pi}{12} \rightarrow \pi-\frac{\pi}{12} \,/\, 2\pi-\frac{\pi}{12}$ $\rightarrow 2k\pi+\underbrace{\pi+\frac{\pi}{6}}_{\frac{7\pi}{6}} \rightarrow x=k\pi+\frac{7\pi}{12} \rightarrow \frac{7\pi}{12} \,/\, \pi+\frac{7\pi}{12}$ } مجموع $=5\pi$ ۲	۸	
$2\cos^2(\alpha)(2\cos^2(2\alpha))(2\cos^2(4\alpha))=\frac{1}{8} \rightarrow \cos^2(\alpha)(\cos^2(2\alpha))(\cos^2(4\alpha))=\frac{1}{64}$ ۰/۷۵	۹	
$f(3x)f(x)=1 \rightarrow \cot(x)=\tan(3x) \rightarrow \tan(\frac{\pi}{2}-x)=\tan_{3x}$ $\frac{\pi}{2}-x=k\pi+3x \rightarrow 4x=\frac{\pi}{2}-k\pi \rightarrow x=\frac{\pi}{8}-\frac{k\pi}{4}$ $\pi+\frac{\pi}{8} \,\vert\, \frac{5\pi}{4}+\frac{\pi}{8} \,\vert\, \frac{6\pi}{4}+\frac{\pi}{8} \,\vert\, \frac{7\pi}{4}+\frac{\pi}{8} \,\vert \rightarrow$ مجموع $\frac{2\,[illegible]\pi}{4}+\frac{2\pi}{4}=4\pi$ ۲	۱۰	

۷) عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \to x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0,1 \to x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \to x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0 \to x = \frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

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۹)

$$1+\cos 2x = 2\cos^2 x \ \star$$

$$\left.\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases}\right\} \to 8\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{}} \cos\alpha \cos 2\alpha \cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha \ \star}$$

$$\underbrace{(\sin\alpha \cos\alpha)}_{\sin 2\alpha} \cos 2\alpha \cos 4\alpha = \pm\frac{\sin\alpha}{8} \to \frac{1}{2}\underbrace{(\sin 2\alpha \cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha} \cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2} \times \frac{1}{2}\underbrace{(\sin 4\alpha \cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \to \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \longrightarrow \begin{cases} 8\alpha = 2k\pi + \alpha \ \text{یا}\ 2k\pi + \pi - \alpha \\ 8\alpha = 2k\pi - \alpha \ \text{یا}\ 2k\pi + \pi + \alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \to x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \to x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \to x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \to x = \frac{7\pi}{9}$$

$$\to \boxed{\max A = \frac{8\pi}{9}}$$

$\star$ A نمی‌تواند برابر خود π باشد چون طبق $\star$ $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!