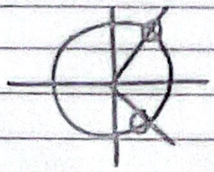
سیرالعلمی

تکلیف ۱۷ فاطمه سلطانی

1) $8\cos x - \tan^2 x = 1$ $[0, 2\pi]$

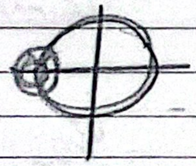
$\hookrightarrow 8\cos x = 1 + \tan^2 x \Rightarrow 8\cos x = \frac{1}{\cos^2 x} \to \cos^3 x = \frac{1}{8}$

$\to \cos x = \frac{1}{2} \to$  $\to$ ۲ جواب

۲) $5\sin^2(x) + 2\cos(3x) = -2$ $[-\pi, \pi]$

$\hookrightarrow \frac{1-\cos 2x}{2}$ $\quad 4\cos^3 x - 3\cos x$

$16\cos^3 x - 10\cos^2 x - 12\cos x + 14 = 0 \to 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

$\div \to \cos x = -1 \to$  $\to x = -\pi, \pi \to$ ۲ جواب

۳) $3\sin(x)\cos(2x) + \sin(x) = 1$ $[0, 2\pi]$

$3\sin x(\cos(2x)) + \sin(x) - 1 = 0 \to 3\sin x(1 - 2\sin^2 x) + \sin(x) - 1 = 0$

$\sin^2 x = \frac{1-\cos 2x}{2} \to \cos 2x = 1 - 2\sin^2 x$

$\leftarrow -6\sin^3 x + 3\sin x + \sin x - 1 = 0$

$\underbrace{-4\sin^3 x + 3\sin x}_{\sin 3x} = 1$ $\hookrightarrow \sin 3x = 1 \to 3x = \frac{\pi}{2} \to x = \frac{\pi}{6}$

طبق فرمول

۴) $\dfrac{1}{\sin\left(\frac{\pi+4x}{2}\right)} + \dfrac{1}{\cos\left(\frac{\pi+8x}{2}\right)} = 0$, $\tan 2\alpha$ کدام؟ ، $\alpha \in [0, \pi]$

↙

$\dfrac{1}{\cos 2x} + \dfrac{1}{-\sin 4x} = \dfrac{1}{\cos 2x} + \dfrac{1}{-2\sin 2x \cos 2x}$

$\sin\left(\frac{\pi}{2}+2x\right)$, $\cos\left(\frac{\pi}{2}+4x\right)$

$\dfrac{-2\sin 2x + 1}{-2\sin 2x \cos 2x} = 0 \rightarrow -2\sin 2x + 1 = 0 \rightarrow \sin 2x = \dfrac{1}{2}$

$\hookrightarrow 2x = \dfrac{\pi}{6} \rightarrow x = \dfrac{\pi}{12}$

$2x = \dfrac{5\pi}{6} \rightarrow x = \dfrac{5\pi}{12}$ $\rightarrow \dfrac{5\pi - \pi}{12} = \dfrac{\pi}{3}$

$\rightarrow \tan 2\times\dfrac{\pi}{3} = \tan \dfrac{2\pi}{3} = \boxed{-\sqrt{3}}$

۵) $m(\cos x - \sin x) - 2\sqrt{6}\sin(2x) = \sqrt{6}$, $\cos\left(x+\frac{\pi}{4}\right) = \dfrac{1}{\sqrt{3}}$

$\rightarrow m = ?$

$\hookrightarrow \cos\left(x+\frac{\pi}{4}\right) = \dfrac{1}{\sqrt{3}} \rightarrow \cos x \cos\dfrac{\pi}{4} - \sin x \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{3}}$

$\dfrac{\sqrt{2}}{2}(\cos x - \sin x) = \dfrac{1}{\sqrt{3}}$

$\rightarrow \underline{\cos x - \sin x} = \dfrac{1}{\sqrt{3}} \times \dfrac{2}{\sqrt{2}} = \dfrac{2}{\sqrt{6}}$

* $(\cos x - \sin x)^2 = \underbrace{\cos^2 x + \sin^2 x}_{1} - 2\sin x \cos x = \dfrac{4}{6} = \dfrac{2}{3}$

$\rightarrow \underline{2\sin x \cos x} = \dfrac{1}{3}$

صفحه بعد

$m(\sin^2 x - \cos^2 x) - 2\sqrt{6}\sin(2x) = \sqrt{6} \longrightarrow$

$\frac{2}{\sqrt{6}}m - 2\sqrt{6}\times\frac{1}{2} = \sqrt{6} \rightarrow \frac{2}{\sqrt{6}}m = 2\sqrt{6} \rightarrow m = 2\sqrt{6}\times\frac{\sqrt{6}}{2}$

$\boxed{m = 6}$

۴) $\sin(x+\frac{\pi}{6})\cos(2x-\frac{\pi}{3}) = 1$ → $[0, 2\pi]$

$(\sin x\cos\frac{\pi}{6} + \sin\frac{\pi}{6}\cos x)(\cos x\cos\frac{\pi}{3} - \sin x\sin\frac{\pi}{3}) = 1$

$(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x)(\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x) = 1$

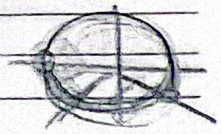
$\frac{\sqrt{3}}{4}\sin x\cos x - \frac{3}{4}\sin^2 x + \frac{1}{4}\cos^2 x - \frac{\sqrt{3}}{4}\sin x\cos x = 1$ ($\sin^2 x = 1-\cos^2 x$)

$\frac{1}{4}\cos^2 x - \frac{3}{4} + \frac{3}{4}\cos^2 x = 1 \rightarrow \cos^2 x = \frac{7}{4} \propto$ غ ق ق

$\boxed{\text{۵ جواب}}$

۷) $\sin x + \sqrt{3}\cos x = \sqrt{2}$ → $\tan\alpha = \sqrt{3}$ $[-\pi, 2\pi]$

$\frac{1}{\sqrt{1+1^2}}\sin(x+\frac{\pi}{3}) = \sqrt{2}$



$2\sin(x+\frac{\pi}{3}) = \sqrt{2} \rightarrow \sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2} \rightarrow$

$x+\frac{\pi}{3} = \frac{\pi}{4} \rightarrow x = \frac{-\pi}{12}$ ✓ $\rightarrow \frac{5\pi}{4} - \frac{\pi}{12} = \boxed{\frac{7\pi}{3}}$

$x+\frac{\pi}{3} = \frac{3\pi}{4} \rightarrow x = \frac{9\pi-4\pi}{12} = \frac{5\pi}{12}$ ✓

$14 + \frac{8}{3} =$

[۰, ۲π]

۸) $۴\sin\alpha \sin\left(\frac{۳\pi}{۴}-\alpha\right)=۱$

$\cos\alpha \rightarrow -۴\sin\alpha\cos\alpha=۱ \rightarrow -۲\sin۲\alpha=۱$

$\rightarrow \sin۲\alpha=-\frac{۱}{۲} \rightarrow ۱)\ ۲\alpha=\frac{۷\pi}{۶} \rightarrow \alpha=\frac{۷\pi}{۱۲}$

$۲)\ ۲\alpha=\frac{۱۱\pi}{۶} \rightarrow \alpha=\frac{۱۱\pi}{۱۲}$

$\frac{۳\pi}{۲}$

۹)

$۱+\cos۴\alpha\cos^۲\left(\frac{\alpha}{۲}\right) \rightarrow ۸\cos^۲\alpha\cos^۲(۲\alpha)\cos^۲(۴\alpha)=\frac{۱}{۸}$?

$|\cos\alpha\cos۲\alpha\cos۴\alpha|=\frac{۱}{۸} \rightarrow \sin۸\alpha=۸\sin\alpha\cos\alpha\cos۲\alpha\cos۴\alpha$

$\frac{\sin۸\alpha}{۸\alpha}=۱ \rightarrow |\sin۸\alpha|<|۸\alpha| \rightarrow \alpha_{max}=\frac{۸\pi}{۹}$

۱۰)

۱۰) $\sin(۳\alpha)\sin(\alpha)=۱ \rightarrow$ ✓ [۰, ۶π]

$۳\alpha=\frac{۸\pi}{۲}-\alpha+۲k\pi \rightarrow ۴\alpha=\frac{۸\pi}{۲}+۲k\pi \rightarrow \alpha=\frac{۹\pi}{۸}, \frac{۱۱\pi}{۸}, \frac{۱۵\pi}{۱۶}, \frac{۱۰\pi}{۸}$

$\rightarrow ۹\times\frac{۶\pi}{۸}=۶\pi \rightarrow$ ۶π