۱۳٫۵

حل تمرین

۱) الف) $\lim_{x \to 1^+} \frac{x^2+2x}{x+1} = \frac{1+2}{1+1} + \frac{1+2}{1+1} = \frac{3}{2} + \frac{3}{2} = 3$

$x = 1^+ \longrightarrow \frac{x^2+2x}{x+1} = \frac{3}{2} = 1.5$ ✓

ب) $\lim_{x \to 1^-} \frac{x^2+2x}{x+1} = 1 + \frac{3}{1+1} = 1 + \frac{3}{2} = 2.5$

$x = 1^- \longrightarrow \frac{x^2+2x}{x+1} = \frac{3}{2} = 1.5$ ✓

۲)

ج) $\lim_{x \to 1} \frac{x^2+2x}{x+1} = 1 + \frac{3}{1+1} = 2.5$

$x \to 1^+ \Rightarrow 1 + \frac{3}{1.0^+} = 3^-$ ✓

$x \to 1^- \Rightarrow 1 + \frac{3}{1.0^-} = 3^+$

د) $\lim_{x \to 0} \frac{x+1}{[x^0]} = \frac{x \to 0^+}{x \to 0^-} \frac{f(0)+1}{[0^+]} = \frac{1}{0} \to \text{نامعین}$

$\frac{f(0)+1}{[0^-]} = \frac{1}{0} \to \text{نامعین}$ ✓

ب) $\lim_{x \to 2^-} [x]^2 - 2 = [2^-]^2 - 2 = 1^2 - 2 = -1$ ✓

ج) $\lim_{x \to 2^-} [5x - 2] = \lim_{x \to 2^-} [10^- - 2] = \lim_{x \to 2^-} [8^-] = 7$ ✓

د) $\left[\lim_{x \to 2^-} 3x - 2\right] = \left[\lim_{x \to 2^-} 3(2^-) - 2\right] = [6^- - 2] = [4^-] = 3$ ✓

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ج) $\lim_{x \to 2^+} [x^3 - 6] = [\lim_{x \to 2^+} x^3 - 6] = [8 - 6] = 2$ ✓

د) $\lim_{x \to 2^-} x[x] + 1 = \lim_{x \to 2^-} x(1) + 1 = 2 + 1 = 3$ ✓

هـ) $\lim_{x \to 1^-} \left[\frac{x}{x+1}\right] = \lim_{x \to 1^-} \left[\frac{1}{2}\right] = \lim_{x \to 1^-} [0] = 0$ ✓

و) $\left[\lim_{x \to 1^-} \frac{x+1}{x}\right] = \left[\lim_{x \to 1^-} \frac{2}{1}\right] = 2$ ✓

ز) $\left[\lim_{x \to 1^+} \frac{x+1}{x}\right] = \left[\lim_{x \to 1^+} \frac{2}{1}\right] = 2$ ✓

ح) $\lim_{x \to 1} \frac{x-1}{(x-1)(x-1)} = \frac{1}{x-1}$

$\lim_{x \to 1^+} \frac{1}{x-1} = \frac{1}{0^+} = +\infty$

$\lim_{x \to 1^-} \frac{1}{x-1} = \frac{1}{0^-} = -\infty$

ط) $\lim_{x \to 3} \frac{3-x}{(x-3)^2}$

$x \to 3^+ : \frac{-1}{0^+} = -\infty$

$x \to 3^- : \frac{+1}{0^+} = -\infty$

$\lim_{x \to 3} \frac{-1}{(x-3)^2} = -\infty$ ✓

ي) $\lim_{x \to 2^-} [-x] - [x] = [-2] - [2] = -4$

$\lim_{x \to 2^-} [-x] - [x] = [-2] - [2] = -4$

$\lim_{x \to 2^+} [-x] - [x] = [-3] - [2] = -5$

$\lim_{x \to 2} = -4 + 5 = 1$

ك) $\lim_{x \to -\frac{1}{2}} \left[\frac{3x}{-1}\right] = \lim_{x \to -\frac{1}{2}} \left[\frac{-3}{1}\right] = [-3] = -3$

سوال ۵

الف) $\lim_{x \to -۳} [-۲x] - [۵x]$

$\lim_{x \to (-۳)^+} [-۲x] - [۵x] = ۸ - (-۱۵) = ۲۳$

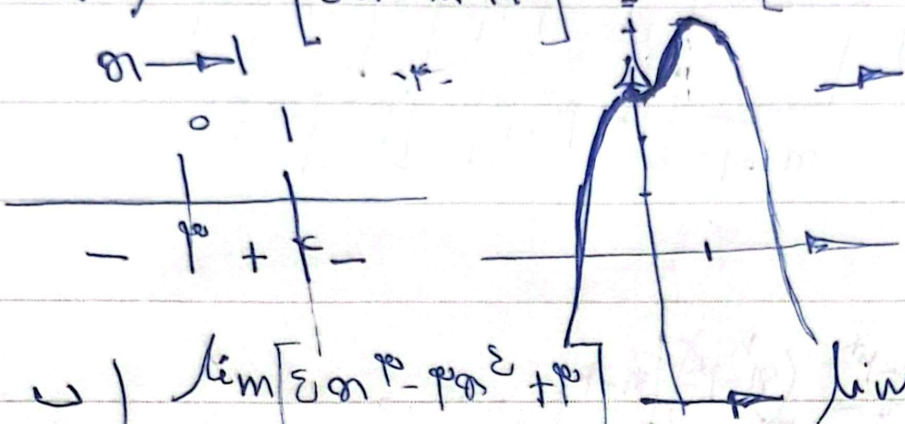
$\lim_{x \to (-۳)^-} [-۲x] - [۵x] = ۹ - (-۱۴) = ۲۵$

ب) $\lim_{x \to (-\frac{۱}{۲})^-} \left[\frac{-۱}{x^۴}\right] = -۱۴$

$x < -\frac{۱}{۲} \to x^۴ > \frac{۱}{۱۶} \to -x^۴ < \frac{-۱}{۱۶} \to -\frac{۱}{x^۴} > -۱۴$

الف) $\lim_{x \to ۱} [۴x^۳ - ۳x^۴ + ۳] \to [۱۲x^۳ - ۱۲x^۳] = [۱۲x^۳(۱-x)]$

$x \to ۱ \to [۴x^۳ - ۳x^۴ + ۳] = [۴] = ۳$

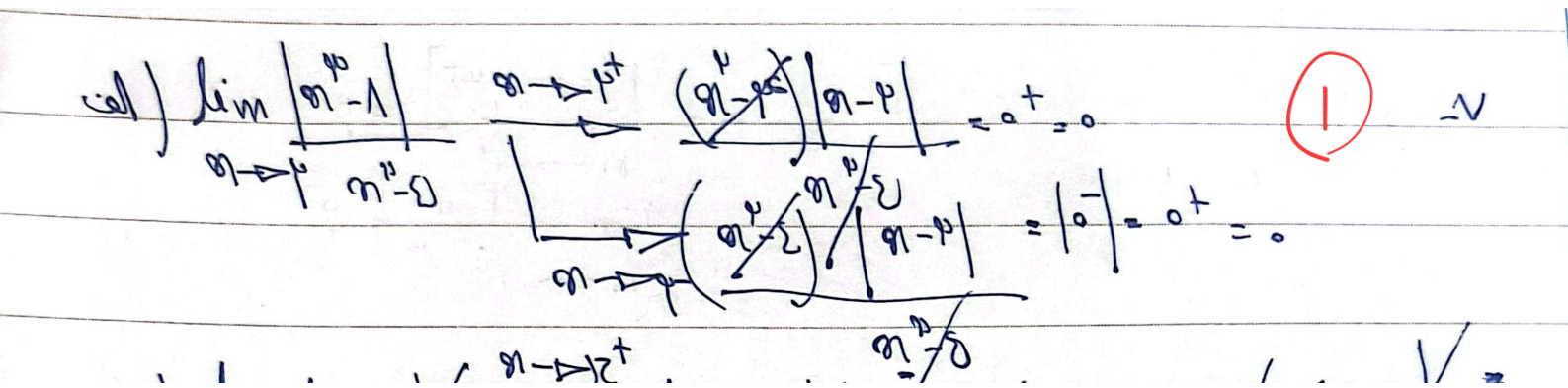


ب) $\lim_{x \to ۰} [۴x^۳ - ۳x^۴ + ۳]$

$\lim_{x \to ۰^+} [۳^+] = ۳$

$\lim_{x \to ۰^-} [۳^-] = ۲$

(۲)



ألف) $\lim_{x \to 2} \frac{|x^3 - 1|}{x^2 - 4}$

x	$-\infty$		1		$+\infty$
$x^3 - 1$		$-$	0	$+$	

$$\lim_{x \to 2^+} \frac{|x^3 - 1|}{x^2 - 4} = \lim_{x \to 2^+} \frac{x^3 - 1}{x^2 - 4} = \lim_{x \to 2^+} \frac{(x-1)(x^2 + x + 1)}{(x-2)(x+2)} = \frac{12}{4} = 3$$

$$\lim_{x \to 2^-} \frac{|x^3 - 1|}{x^2 - 4} = \lim_{x \to 2^-} \frac{-(x^3 - 1)}{x^2 - 4} = \lim_{x \to 2^-} \frac{-(x-1)(x^2 + x + 1)}{(x-2)(x+2)} = \frac{-12}{4} = -3$$

ب) $\lim_{x\to 0} \sqrt{x-2\sqrt{x}}$ = تعریف نشده $x\geq 0$ (I)

$x-2\sqrt{x}\geq 0 \to \sqrt{x}(\sqrt{x}-2)\geq 0 \to$ $\begin{array}{c|ccc} & 0 & & 4 \\ \hline \sqrt{x} & 0 & + & + \\ \sqrt{x}-2 & - & - & + \end{array}$ $\to x\geq 4$ یا $x\leq 0$ (II)

دامنه : $x=0$ یا $x\geq 4$ $\to$ (I)∩(II) مبدا تابع در نقاط اطراف صفر تعریف نمی شود.

الف) $\lim_{x\to \frac{3\pi}{2}} \sqrt{\cos x}$ $\xrightarrow{x\to \frac{3\pi}{2}^+}$ $\lim \sqrt{\cos \frac{3\pi}{2}^+} = \sqrt{0^+} = 0$ ✓

$x\to \frac{3\pi}{2}^-$ $\to \sqrt{\cos \frac{3\pi}{2}^-} = \sqrt{0^-} \to$ تعریف نشده

ب) $\lim_{x\to 0} \sqrt{x^2-2\sqrt{x}}$ $\xrightarrow{0^+}$ $\sqrt{0^+-2\sqrt{0^+}} = 0$

$0^- \to \sqrt{0^--2\sqrt{0^-}} \to$ تعریف نشده

ج) $\lim_{x\to 0^+} (-1)^{\frac{[\sin x]}{x^3-x^2}}$ $\xrightarrow{x\to 0^+}$ $\lim (-1)^{\frac{[0^+]}{x^3-x^2}} = (-1)^{\frac{0}{x^2(x-1)}}$

$\lim_{x\to 0^+} \frac{1}{x^3-x^2} = \frac{1}{-x^2+x^3} = \frac{1}{x^2(x-1)} = \frac{1}{-x^2+x} = \frac{1}{x-x^2} \Rightarrow -\infty$ ✓

$x=\frac{1}{10}$ $\frac{1}{1000}$ $-\frac{1}{100}$

د) $\lim_{x\to 2} \frac{x^2-[x^2]}{x-[x]}$ $\xrightarrow{2^+}$ $\frac{4-4}{2-2} = \frac{0}{0}$

هـ) $\lim_{x\to 2} (x+[x])^{\frac{x-[x]}{x-[x]}} \Rightarrow$ $\xrightarrow{x\to 2^+}$ $2^+ + 2 = 4$ ✓

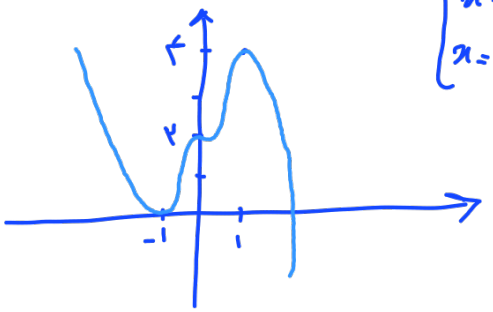
$x\to 2^-$ $\to 2^- + [2^-] = 2 + 1 = 3$

$\lim_{x\to 2^-} \frac{x^2-3}{x-1} = \frac{4-3}{2-1} = \frac{1}{1} = 1$

الف) $y = 2x^3 - 3x^2 + 1$

$y' = 6x^2 - 6x \xrightarrow{y'=0} 6x(x-1) = 0 \rightarrow 6x^2(x-1)(x+1) = 0 \rightarrow \begin{cases} x = 0 \\ x = 1 \\ x = -1 \end{cases}$

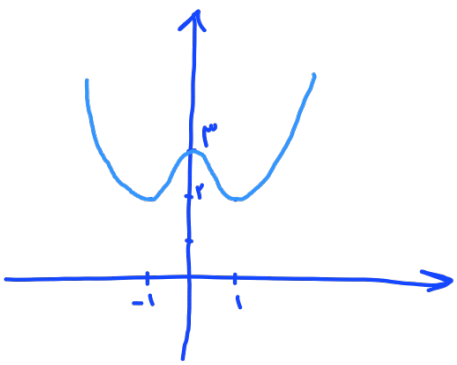
x	$-\infty$		-1		0		1		$+\infty$
y'		$-$	0	$+$	0	$+$	0	$-$	
y		↘	-2	↗	1	↗	0	↘	



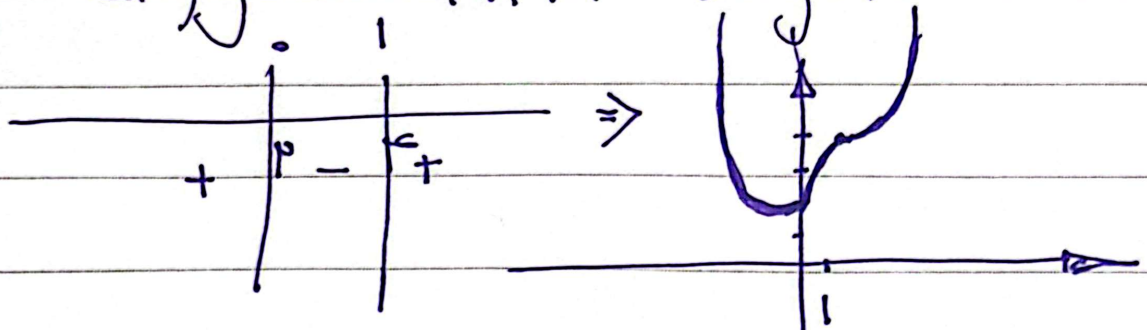
ب) $y = x^4 - 2x^2 + 1$

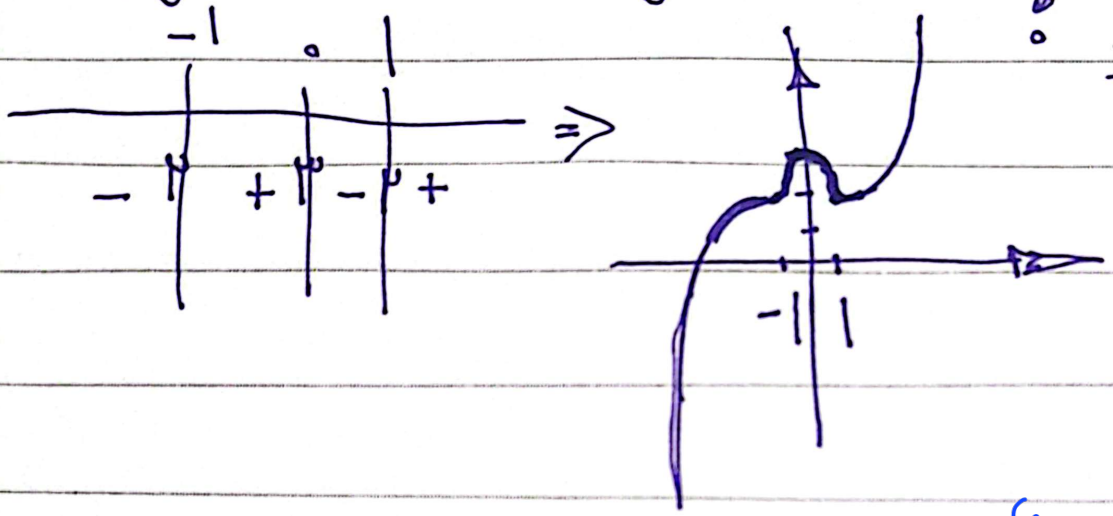
$y' = 4x^3 - 4x \xrightarrow{y'=0} 4x(x^2-1) = 0 \rightarrow 4x(x-1)(x+1) = 0 \rightarrow \begin{cases} x = 0 \\ x = 1 \\ x = -1 \end{cases}$

x	$-\infty$		-1		0		1		$+\infty$
y'		$-$	0	$+$	0	$-$	0	$+$	
y		↘	0	↗	1	↘	0	↗	



ج) $y = 2x^3 - 3x^2 + 1 \rightarrow y' = 6x^2 - 6x = 6x(x-1)$



د) $y = x^4 + 4x^3 - 3x + 1 \rightarrow y' = 4x^3 + 12x^2 - 3 = 3(4x^3 + 4x^2 - 1)$

$4x^3 + 4x^2 - 1 = 0 \rightarrow (x+1)(4x^2 - 1) = 0 \rightarrow \begin{cases} x = -1 \\ x = \frac{1}{2} \\ x = -\frac{1}{2} \end{cases}$