نرگس زنده دل

۱) الف) $\lim\limits_{x \to 2^+} \frac{2x+5}{x+1} = \frac{2x+2}{x+1} + \frac{3}{x+1} = 2 + \frac{3}{x+1} = 3 = 2 + \frac{3}{2^+ + 1} = 2 + \frac{3}{3^+} = 2 + 1^- = 3^-$

$x = 2.1 \longrightarrow \frac{4.2+5}{3.1} = \frac{9.2}{3.1} = 3^-$

ب) $\lim\limits_{x \to 2^-} \frac{2x+5}{x+1} = 2 + \frac{3}{x+1} = 2 + \frac{3}{3^-} = 2 + 1^+ = 3^+$

$x = 1.9 \longrightarrow \frac{3.8+5}{2.9} = \frac{8.8}{2.9} = 3^+$

ج) $\lim\limits_{x \to 2} \frac{2x+5}{x+1} = 2 + \frac{3}{x+1}$

$\xrightarrow{x \to 2^+} 2 + \frac{3}{3^+} = 3^-$

$\xrightarrow{x \to 2^-} 2 + \frac{3}{3^-} = 3^+$

$\Rightarrow 3$ $\qquad \frac{2(2)+5}{3} = \frac{9}{3} = 3$

د) $\lim\limits_{x \to 0} \frac{2x+7}{[x]}$

$\xrightarrow{x \to 0^+} \frac{2(0)+7}{[0^+]} = \frac{7}{0} \Rightarrow$ ت.ن

$\xrightarrow{x \to 0^-} \frac{2(0)+7}{[0^+]} = \frac{7}{0} \Rightarrow$ ت.ن

الف) $\lim\limits_{x \to 3^-} 4[x] - 2 = 4[3^-] - 2 = 4 \times 2 - 2 = 6$

ب) $\lim\limits_{x \to 3^-} [4x - 2] = [\lim\limits_{x \to 3^-} 12^- - 2] = [\lim\limits_{x \to 3^-} 10^-] = 9$

ج) $\left[\lim\limits_{x \to 3} 4x - 2\right] = \left[\lim\limits_{x \to 3^-} 4(3) - 2\right] = 10$

د) $\left[\lim_{x\to 3^+} 4x-2\right] = \left[\lim_{x\to 3^+} 3(4)-2\right] = 10$

الف) $\lim_{x\to 3^-} \frac{3[x]+1}{2} = \lim_{x\to 3^-} \frac{3(2)+1}{2} = \frac{7}{2}$ ۳)

ب) $\lim_{x\to 3^-} \left[\frac{3x+1}{2}\right] = \lim_{x\to 3^-} \left[\frac{10^-}{2}\right] = \lim_{x\to 3^-} \left[5^-\right] = 4$

ج) $\left[\lim_{x\to 3^-} \frac{3x+1}{2}\right] = \left[\lim_{x\to 3^-} \frac{10}{2}\right] = 5$

د) $\left[\lim_{x\to 3^+} \frac{3x+1}{2}\right] = \left[\lim_{x\to 3^+} \frac{10}{2}\right] = 5$

الف) $\lim_{x\to 1} \frac{\cancel{x-5}}{\cancel{(x-5)}(x-1)} = \frac{1}{x-1} = \frac{1}{2-1} = 1$ ۴)

ب) $\lim_{x\to 3} \frac{x-4}{(x-3)^2}$ $\begin{cases} \xrightarrow{x\to 3^+} \frac{-1}{0^+} = -\infty \\ \xrightarrow{x\to 3^-} \frac{-1}{0^+} = -\infty \end{cases}$ حد دارد

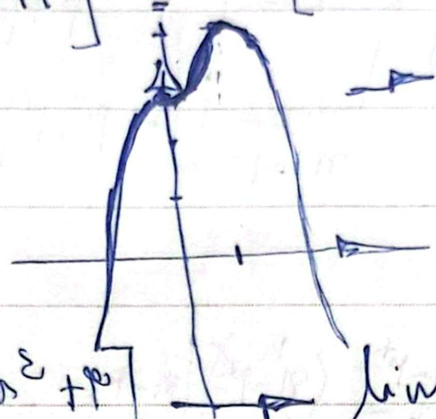
الف) $\lim_{x\to -3} [-3x]-[5x]$ $\begin{cases} \xrightarrow{x\to -3^-} \lim_{x\to -3^-} \left[-3(-3^-)\right] - \left[5(-3^-)\right] = 9+16 = 25 \\ \xrightarrow{x\to -3^+} \lim \left[-3(-3^+)\right] - \left[5(-3^+)\right] = 8+15 = 23 \end{cases}$ ۵)

ب) $\lim_{x\to \left(-\frac{1}{2}\right)^-} \left[\frac{-1}{x^4}\right] \xrightarrow{x\to -\frac{1}{2}^-} \lim_{x=-\frac{2}{3}} \left[\frac{-1}{\frac{16}{81}}\right] = \lim_{x\to \left(-\frac{1}{2}\right)} \left[-6^-\right] = -7$

$-\frac{81}{16}$

الف) $\lim\limits_{x \to 1} [۴x^{۳} - ۳x^{۴} + ۲] \longrightarrow [۱۲x^{۲} - ۱۲x^{۳}] = [۱۲x^{۲}(۱-x)]$

	۰		۱	
−	۰	+	۰	−



$\longrightarrow \lim\limits_{x \to 1} [۴x^{۳} - ۳x^{۴} + ۲] = [۳^{-}]$
$= ۲$

ب) $\lim\limits_{x \to ۰} [۴x^{۳} - ۳x^{۴} + ۲] \longrightarrow \lim\limits_{x \to ۰^{+}} [۲^{+}] = ۲$

$\lim\limits_{x \to ۰^{-}} [۲^{-}] = ۱$

۷-

الف) $\lim\limits_{x\to ۲} \dfrac{|x^۳-۸|}{x^۲-۴}$ $\quad x\to ۲^+ \Rightarrow \dfrac{(\cancel{x^۲-۴})\,|x-۲|}{\cancel{x^۲-۴}} = ۰^+ = ۰$

$\quad x\to ۲^- \Rightarrow \dfrac{(\cancel{x^۲-۴})\,|x-۲|}{\cancel{x^۲-۴}} = |۰^-| = ۰^+ = ۰$

$x\to ۲^+$

7) $\lim\limits_{x \to \pi} \dfrac{|\sin x|}{\sin 2x} = \dfrac{|\sin x|}{2\sin x \cos x} \xrightarrow{x \to \pi^+} \dfrac{-\cancel{\sin \pi^+}}{2\cancel{\sin \pi^+}\cos \pi^+} = \dfrac{1}{2}$

$\xrightarrow{x \to \pi^-} \dfrac{\cancel{\sin \pi}}{2\cancel{\sin \pi}\cos \pi} = -\dfrac{1}{2}$

(۱ الف) $\lim_{x\to 3\pi/2} \sqrt{\cos x} \xrightarrow{3\pi/2^+} \lim_{x\to 3\pi/2^+} \sqrt{\cos 3\pi/2^+} = \sqrt{0^+} = 0$

$\xrightarrow{3\pi/2^-} \sqrt{\cos 3\pi/2^-} = \sqrt{0^-} \to$ ندارد

ب) $\lim_{x\to 0} \sqrt{x-2\sqrt{x}} \xrightarrow{0^+} \sqrt{0^+-2\sqrt{0^+}} = 0$

$\xrightarrow{0^-} \sqrt{0^- - 2\sqrt{0^-}} \to$ ندارد

(۴ الف) $\lim_{x\to 0^+} \dfrac{(-1)^{[\sin x]}}{x^3-x^2} \longrightarrow \lim_{x\to 0^+} \dfrac{(-1)^{[0^+]}}{x^3-x^2} = \dfrac{1}{x^3-x^2}$ $\quad x^3-x^2 \to 0^-$ $\quad x^2(x-1)$

$\lim_{x\to 0^+} \dfrac{1}{x^3-x^2} = \dfrac{1}{-9/1000} = \dfrac{1000}{-9} \Rightarrow -\infty$

$x = 1/10 \quad 1/1000 - 1/100$

ب) $\lim_{x\to 2} \dfrac{x^2-[x^2]}{x-[x]} \xrightarrow{2^+} \dfrac{4^+-4}{2^+-2} =$

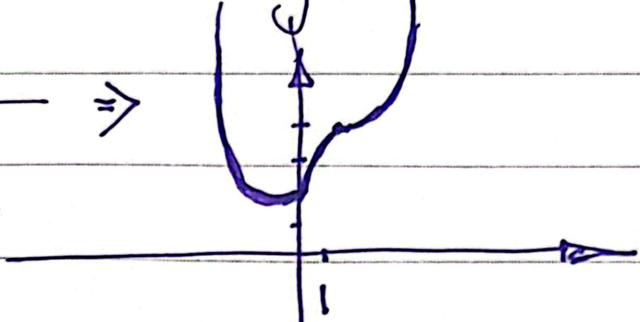
ب) $\lim_{x\to 2} (x+[x]) \propto \left\{ \dfrac{x-[x]}{x-[x]} \right. \Rightarrow \xrightarrow{x\to 2^+} 2^+ + 2 = 4$

$\xrightarrow{x\to 2^-} 2^- + [2^-] = 2+1 = 3$

الف) $y = 5x^3 - 3x^5 + 3 \longrightarrow y' = 15x^2 - 15x = 15x(x-1)$ ۱۰-

x		۰		۱	
y'	+		−		+

$\Rightarrow$



ب) $y = x^4 - 2x^2 + 3 \longrightarrow y' = 4x^3 - 4x = 4x(x^2-1)$

$4x \to 0$, $(x^2-1) \to \pm 1$

x		−۱		۰		۱	
y'	−		+		−		+

$\Rightarrow$