

٢٠ أدلة مفروضا

١. الف) $\lim_{x \rightarrow p^+} \frac{p(x)+0}{x+1} \rightarrow \frac{p(p)+0}{p+1} = \frac{9}{p} = p$ ✓

ب) $\lim_{x \rightarrow p^-} \frac{p(x)+0}{x+1} \rightarrow \frac{p(p)+0}{p+1} = \frac{9}{p} = p$ ✓

ج) $\lim_{x \rightarrow p} \frac{p(x)+0}{x+1} \rightarrow \frac{p(p)+0}{p+1} = \frac{9}{p} = p$ } $\lim_{x \rightarrow p} \frac{p(x)+0}{x+1} = p$ ✓

د) $\lim_{x \rightarrow 0} \frac{p(x)+v}{[x^2]} \rightarrow \frac{p(0)+v}{[0^+]} = \infty$ ✓
 $\rightarrow \frac{p(0)+v}{[0^-]} = -\infty$

٢. الف) $\lim_{x \rightarrow p^-} f(x) - p = f(p^-) - p = f(p) - p = 1 - p = 4$ ✓

ب) $\lim_{x \rightarrow p^-} [f(x) - p] = [f(p^-) - p] = [1^- - p] = [1 - p] = 4$ ✓

ج) $[\lim_{x \rightarrow p^-} f(x) - p] = [f(p^-) - p] = [1 - p] = 4$ ✓

د) $[\lim_{x \rightarrow p^+} f(x) - p] = [f(p^+) - p] = [1^+ - p] = 4$ ✓

٣. الف) $\lim_{x \rightarrow p^-} \frac{p(x)+1}{p} = \frac{p(p^-)+1}{p} = \frac{p(p)+1}{p} = \frac{v}{p}$ ✓

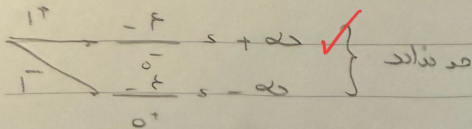
ب) $\lim_{x \rightarrow p^-} \left[\frac{p(x)+1}{p} \right] = \left[\frac{p(p^-)+1}{p} \right] = \left[\frac{1^-+1}{p} \right] = \left[\frac{2^-}{p} \right] = \frac{2}{p}$ ✓

ج) $\left[\lim_{x \rightarrow p^-} \frac{p(x)+1}{p} \right] = \left[\frac{p(p^-)+1}{p} \right] = \left[\frac{1^-+1}{p} \right] = \frac{2}{p}$ ✓

د) $\left[\lim_{x \rightarrow p^+} \frac{p(x)+1}{p} \right] = \left[\frac{p(p^+)+1}{p} \right] = \left[\frac{1^++1}{p} \right] = \frac{2}{p}$ ✓

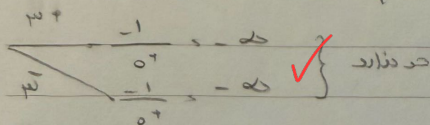
الف) $\lim_{x \rightarrow 0} \frac{x-0}{x^2-4x+0} = \frac{1-0}{1-4+0} = \frac{-1}{-3}$

$\hookrightarrow (x-1)(x-0) \frac{1}{x-4} = \frac{0}{-3}$



ب) $\lim_{x \rightarrow 3} \frac{x-3}{x^2-4x+9} = \frac{3-3}{9-12+9} = \frac{0}{6}$

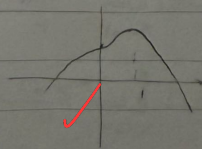
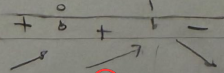
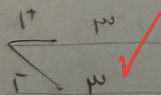
$\hookrightarrow (x-3)^2 \frac{1}{x+6}$



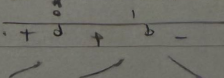
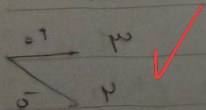
ج) $\lim_{x \rightarrow -3} [-3x] - [0x] = [9] - [10] = 1 - (-10) = 11$

د) $\lim_{x \rightarrow (-1)} \left[\frac{-1}{x^2} \right] = [-14] = -14$

هـ) $\lim_{x \rightarrow 1} [x^3 - 3x^2 + 3x] = [1^3 - 1^2 + 1] = [1 - 1 + 1] = 1$



و) $\lim_{x \rightarrow 0} [x^3 - 3x^2 + 3x] = [0^3 - 0^2 + 0] = [0 - 0 + 0] = 0$



الف) $\lim_{x \rightarrow p} \frac{x^p - 1}{x^p - 1}$

$\begin{aligned} & \xrightarrow{p^+} \frac{x^p - 1}{x^p - 1} = \frac{(x-1)(x^{p-1} + x^{p-2} + \dots + 1)}{(x-1)(x^{p-1} + x^{p-2} + \dots + 1)} = \frac{1}{1} = 1 \end{aligned}$

$\xrightarrow{p^-} \frac{1 - x^p}{1 - x^p} = \frac{-(x-1)(x^{p-1} + x^{p-2} + \dots + 1)}{-(x-1)(x^{p-1} + x^{p-2} + \dots + 1)} = \frac{1}{1} = 1$

محدود

الف) $\lim_{x \rightarrow \pi} \frac{|\sin x|}{\sin x}$

$\xrightarrow{\pi^+} \frac{-\sin x}{\sin x} = \frac{-1}{1} = -1$

$\xrightarrow{\pi^-} \frac{\sin x}{\sin x} = \frac{1}{1} = 1$

محدود

الف) $\lim_{x \rightarrow \frac{\pi}{2}} \sqrt{\cos x}$

$\xrightarrow{\frac{\pi}{2}^+} \sqrt{\cos x} = \sqrt{0} = 0$

$\xrightarrow{\frac{\pi}{2}^-} \sqrt{\cos x} = \sqrt{0} = 0$

محدود

الف) $\lim_{x \rightarrow 0} \sqrt{x - \sqrt{5x}}$

$x - \sqrt{5x} \geq 0 \Rightarrow \sqrt{x}(\sqrt{x} - \sqrt{5}) \geq 0$

$\sqrt{x} \geq \sqrt{5} \Rightarrow x \geq 5$

محدود

الف) $\lim_{x \rightarrow 0^+} \frac{(-1)^{[\sin x]}}{x^p - x^q}$

$\frac{(-1)^0}{x^p - x^q} = \frac{1}{x^p - x^q} = \frac{1}{x^p(1 - x^{q-p})} \rightarrow \infty$

محدود

الف) $\lim_{x \rightarrow p} \frac{x^p - [p]}{x - [p]}$

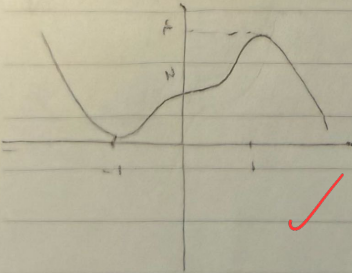
$\xrightarrow{p^+} \frac{x^p - p}{x - p} = \frac{(x-p)(x^{p-1} + x^{p-2} + \dots + 1)}{x - p} = x^{p-1} + x^{p-2} + \dots + 1 \rightarrow p^{p-1} + p^{p-2} + \dots + 1$

$\xrightarrow{p^-} \frac{x^p - p}{x - p} = \frac{(x-p)(x^{p-1} + x^{p-2} + \dots + 1)}{x - p} = x^{p-1} + x^{p-2} + \dots + 1 \rightarrow p^{p-1} + p^{p-2} + \dots + 1$

محدود

$$y = 0x^3 - 13x^0 + 12 = 10x^2 - 10x^2 + 10x^2(1 - x^2)$$

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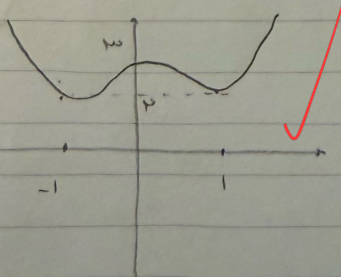
$$\begin{array}{c} -1 \quad 0 \quad +1 \\ - \quad + \quad + \quad - \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \end{array}$$

$$x=1 \rightarrow y = 0 - 13 + 12 = -1$$

$$x=0 \rightarrow y = 12$$

$$x=-1 \rightarrow y = 0 + 13 + 12 = 25$$

$$y = 2x^2 - 2x^2 + 12 = 2x^3 - 2x = 2x(x^2 - 1)$$



$$\begin{array}{c} -1 \quad 0 \quad +1 \\ - \quad + \quad - \quad + \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \end{array}$$

$$x=1 \rightarrow y = 0$$

$$x=0 \rightarrow y = 12$$

$$x=-1 \rightarrow y = 0$$