

$$f(x) = 2 \left[ \frac{x-1}{2} \right] + a \left[ \frac{x+1}{2} \right] \Rightarrow \lim_{x \rightarrow -1} 2 \left[ \frac{x-1}{2} \right] + a \left[ \frac{x+1}{2} \right]$$

$$\left[ 1 - \frac{x}{2} \right] = 1 + \left[ -\frac{x}{2} \right]$$

$$\hookrightarrow -1^+ \Rightarrow 2 + 2 \left[ -\frac{-1^+}{2} \right] + a \left[ \frac{-1^++1}{2} \right] = 2 + 2[1^-] + a[0^+] = 2 + 2(0) + a(0) = 2$$

$$\hookrightarrow -1^- \Rightarrow 2 + 2 \left[ -\frac{-1^-}{2} \right] + a \left[ \frac{-1^-+1}{2} \right] = 2 + 2[1^+] + a[0^-] = 2 + 2 + a(-1) = 2 - a$$

$$2 - a = 2 \Rightarrow a = 2 \Rightarrow \left[ \frac{a}{2} \right] = \left[ \frac{2}{2} \right] = 1$$

$$\lim_{x \rightarrow \frac{\pi}{4}} [2 \sin x - 1]$$

$$x \rightarrow \left( \frac{\pi}{4} \right)^-$$

$$\sin \left( \frac{\pi}{4} \right)^- = \frac{1}{\sqrt{2}}^- \Rightarrow [2 \sin x - 1] = [1^- - 1] = [0^-] = -1$$

$$\lim_{x \rightarrow -\frac{1}{2}} [12x^2 - x] \Rightarrow 12x^2 - x = 12 \left( -\frac{1}{2} \right)^2 - \left( -\frac{1}{2} \right) = 12 \left( \frac{1}{4} \right) + \frac{1}{2} = -1 + \frac{1}{2} = -\frac{1}{2} \notin \mathbb{Z}$$

$$\Rightarrow \left[ -\frac{1}{2} \right] = -1$$

$$\lim_{x \rightarrow \left( \frac{1}{2} \right)^-} \frac{10x - 2 + \left[ \frac{x}{2x^2} \right]}{14x - \left[ -\frac{x}{2x^2} \right]}$$

$$x \rightarrow \left( \frac{1}{2} \right)^- \Rightarrow x^2 > \frac{1}{2} \Rightarrow \frac{1}{x^2} < 2$$

$$\frac{x}{x^2} < 2 \Rightarrow \left[ \frac{x}{x^2} \right] = 1 \quad \frac{x}{x^2} > -1 \Rightarrow \left[ -\frac{x}{x^2} \right] = -1$$

$$x = \frac{1}{2}^- \Rightarrow \frac{10 \left( \frac{1}{2} \right)^- + 9}{14 \left( \frac{1}{2} \right)^- + 1} = \frac{-2 + 9}{-7 + 1} = \frac{1}{-6} = -\frac{1}{6}$$

$$\lim_{n \rightarrow 1^+} \frac{\sin^2 \pi n}{[n] + \cos \pi n} = \lim_{n \rightarrow 1^+} \frac{\sin^2 \pi n}{\cos \pi n + 1} = \frac{0}{-1+1} = \frac{0}{0} \xrightarrow{\text{ل'Hôpital}} \frac{\sin^2 \pi n}{\cos \pi n + 1} = \frac{1 - \cos^2 \pi n}{\cos \pi n + 1} = \frac{(1 - \cos \pi n)(1 + \cos \pi n)}{1 + \cos \pi n}$$

$$[1^+] = 1$$

$$\Rightarrow \lim_{n \rightarrow 1^+} 1 - \cos \pi n = 1 - (-1) = 2$$

$$\lim_{x \rightarrow -1} \frac{\sqrt{2x+3} - \sqrt{2x+5}}{1+\sqrt{x}} = \frac{1-1}{1+(-1)} = \frac{0}{0} \quad \text{رفع ابدا}$$

$$\frac{\sqrt{2x+3} - \sqrt{2x+5}}{1+\sqrt{x}} \times \frac{\sqrt{2x+3} + \sqrt{2x+5}}{\sqrt{2x+3} + \sqrt{2x+5}} \times \frac{\sqrt{x^2 - \sqrt{x}} + 1}{\sqrt{x^2 - \sqrt{x}} + 1} = \frac{-(x+1)}{1+x} \times \frac{1}{2} = \boxed{-\frac{1}{2}}$$

$$\lim_{x \rightarrow 1} \frac{2x - \sqrt{x} + 2}{2x - \sqrt{2x+1}} = \frac{2-1+2}{2-2} = \frac{0}{0} \rightarrow \text{رفع ابدا}$$

$$\frac{2x - \sqrt{x} + 2}{2x - \sqrt{2x+1}} \times \frac{2x + \sqrt{x} + 1}{2x + \sqrt{x} + 1} = \frac{(2x - \sqrt{x} + 2)(2x + \sqrt{x} + 1)}{4x^2 - x - 1} \xrightarrow{x=1} \frac{2(1-\frac{1}{2}+2)(1+\frac{1}{2}+1)}{4-1-1} = \frac{2(1-\frac{1}{2}) \times 2}{2} = \boxed{\frac{1}{2}}$$

از اینجا به بعد از معادله اول استفاده کنیم. (برای رفع ابدا!)

$$\lim_{x \rightarrow 0} \frac{a + \sqrt{bx+c}}{x} = \frac{1}{c} \Rightarrow \lim_{x \rightarrow 0} \frac{a + \sqrt{c}}{0} = \frac{1}{c}$$

$$\hookrightarrow \frac{0}{0} \Rightarrow a + \sqrt{c} = 0 \Rightarrow \sqrt{c} = -a \Rightarrow \boxed{c = a^2}$$

$$\text{Hop} \Rightarrow \frac{b}{\sqrt{bx+c}} = \frac{1}{c} \xrightarrow{x=0} \frac{b}{\sqrt{c}} = \frac{1}{c} \Rightarrow \sqrt{c} = \frac{b}{c} \Rightarrow \sqrt{c} = -a \Rightarrow \boxed{b = -\frac{a}{c}} \rightarrow \frac{ab}{c} = \frac{a(-\frac{a}{c})}{a^2} = \boxed{-\frac{1}{c}}$$

$$\lim_{x \rightarrow -\pi} \frac{f + K[\frac{x}{\pi}]}{\sin x} = +\infty$$

$$x \rightarrow (-\pi)^+ \Rightarrow \frac{f + K[\frac{-\pi}{\pi}]}{\sin(-\pi)^+} = \frac{f - K}{0^+} = +\infty \Rightarrow f - K > 0 \Rightarrow f > K \Rightarrow K < f$$

$$x \rightarrow (-\pi)^- \Rightarrow \frac{f + K[\frac{-\pi}{\pi}]}{\sin(-\pi)^-} = \frac{f - K}{0^-} = +\infty \Rightarrow f - K < 0 \Rightarrow f < K \Rightarrow K > f$$

$\boxed{[-K] = -f}$

$$\lim_{x \rightarrow 1} \frac{b\sqrt{x+\sqrt{x}} - \sqrt{b}}{ax-b} \xrightarrow{\text{Hop}} \frac{b\sqrt{1+\sqrt{1}} - \sqrt{b}}{a-b} = 0 \Rightarrow \frac{b\sqrt{2} - \sqrt{b}}{a-b} = 0 \Rightarrow \boxed{a=b}$$

$$\frac{b\sqrt{x+\sqrt{x}} - \sqrt{b}}{ax-b} \times \frac{b\sqrt{x+\sqrt{x}} + \sqrt{b}}{b\sqrt{x+\sqrt{x}} + \sqrt{b}} = \frac{b^2(x+\sqrt{x}) - b^2}{b^2ax - b^2} \xrightarrow{\text{Hop}} \frac{\frac{1}{2}b^2x^{-\frac{1}{2}}}{\frac{1}{2}b^2x^{-\frac{1}{2}}} = \frac{1}{2}$$