آیلا منوچهرنیا

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۱. $f(x)=2\left[\frac{2-x}{2}\right]+a\left[\frac{x+2}{3}\right]$

$-2^+ \to 2(2)-a=4-a$

$-2^- \to 2(1)+0=2$

$4-a=2 \to a=2$ $\quad \left[\frac{2}{3}\right]=0$

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۲. $\lim\limits_{x\to(\frac{\pi}{6})^-}[2\sin x-1] \to [2(\frac{1}{2})^- -1]=[1^- -1]=[0^-]=-1$

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۳. $\lim\limits_{x\to-\frac{1}{2}}[8x^3-x]=[8(-\frac{1}{2})^3+\frac{1}{2}]=[8(\frac{-1}{8})+\frac{1}{2}]=[-1+\frac{1}{2}]=[\frac{-1}{2}]=-1$

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$x<-\frac{1}{2}\to x^2>\frac{1}{4}\to\frac{1}{x^2}<4\to\frac{3}{x^2}<12\to 11$

۴. $\lim\limits_{x\to(\frac{-1}{2})^-}\frac{10x-5+[\frac{3}{x^2}]}{16x-[-\frac{2}{x^2}]}$

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$x<\frac{-1}{2}\to x^2>\frac{1}{4}\to\frac{1}{x^2}<4\to-\frac{1}{x^2}>-4\to\frac{-2}{x^2}>-8\to-8$

$\frac{10(\frac{-1}{2})^-+6}{16(-\frac{1}{2})^-+8}=\frac{1}{0^-}=-\infty$

۵. $\lim\limits_{x\to1^+}\frac{\sin^2\pi x}{[x]+\cos\pi x}=\frac{\sin^2\pi x}{1+\cos\pi x}=\frac{1-\cos^2\pi x}{1+\cos\pi x}=\frac{(1-\cos\pi x)\cancel{(1+\cos\pi x)}}{\cancel{1+\cos\pi x}}$

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$=1-\cos\pi x\to1-\cos\pi=1-(-1)=2$

۶. $\lim\limits_{x\to-1}\frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}}\times\frac{1+\sqrt[3]{x^2}-\sqrt[3]{x}}{1+\sqrt[3]{x^2}-\sqrt[3]{x}}\times\frac{\sqrt{2x+3}+\sqrt{3x+4}}{\sqrt{2x+3}+\sqrt{3x+4}}=$

$\frac{-\cancel{(1+x)}(1+\sqrt[3]{x^2}-\sqrt[3]{x})}{\cancel{(1+x)}(\sqrt{2x+3}+\sqrt{3x+4})}=\frac{-(1+\sqrt[3]{x^2}-\sqrt[3]{x})}{\sqrt{2x+3}+\sqrt{3x+4}}=\frac{-(1+1+1)}{2}=\frac{-3}{2}$

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۷.
$$\lim_{x\to 1}\frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}}=\frac{2(x-\frac{7}{2}\sqrt{x}+\frac{5}{2})}{2x-\sqrt{3x+1}}=\frac{2(\sqrt{x}-1)(\sqrt{x}-\frac{5}{2})}{2x-\sqrt{3x+1}}$$

$$\times\frac{2x+\sqrt{3x+1}}{2x+\sqrt{3x+1}}=\frac{\cancel{2(\sqrt{x}-1)}(\sqrt{x}-\frac{5}{2})(2x+\sqrt{3x+1})}{4x^2-3x-1}=\frac{(-\frac{3}{2})(4)}{2(2)(\frac{5}{4})}=\frac{-3}{\frac{5}{2}}=\frac{-6}{5}$$

$$4x^2-3x-1\sim 4(x^2-\frac{3}{4}x-\frac{1}{4})\to (x-1)(x+\frac{1}{4})$$

$\sqrt{x}+1$

2

۸.
$$\lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x}=\frac{1}{4}\to\lim\frac{a+\sqrt{c}}{0}=\frac{1}{4}$$

$a<0$

$$\frac{0}{0}\to a+\sqrt{c}=0\to\sqrt{c}=-a\to c=a^2$$

$$\to\frac{\frac{b}{2\sqrt{bx+c}}}{1}=\frac{1}{4}\xrightarrow{x=0}\frac{b}{2\sqrt{c}}=\frac{1}{4}\to 2b=\sqrt{c}\to 2b=-a,\quad -a=\sqrt{c},\quad b=\frac{-a}{2}$$

$$\to\frac{ab}{c}=\frac{a(\frac{-a}{2})}{a^2}=\frac{-a^2}{2a^2}=\frac{-1}{2}$$

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۹.
$$\lim_{x\to -2\pi}\frac{4+k[\frac{x}{\pi}]}{\sin x}=+\infty$$

$$\xrightarrow{-2\pi^+}\frac{4+k[-2^+]}{\sin(-2\pi)^+}=\frac{4-2k}{0^+}=+\infty\to 4-2k>0,\quad 4>2k\to k<2$$

$$\xrightarrow{-2\pi^-}\frac{4+k[-2^-]}{\sin(-2\pi)^-}=\frac{4-3k}{0^-}=+\infty\to 4-3k<0\to 3k>4\to k>\frac{4}{3}$$

$$-2<-k<\frac{-4}{3}\to[-k]=-2$$

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۱۰.
$$\lim_{x\to 8}\frac{b\sqrt{2+\sqrt[3]{x}}-2b}{ax-b}\sim 2b-2b=0\to 2a-b=0\to b=2a$$

$8a-b=0\to b=8a$

$$\times\frac{b\sqrt{2+\sqrt[3]{x}}+2b}{b\sqrt{2+\sqrt[3]{x}}+2b}=\frac{2b^2+b^2x^{\frac{1}{3}}-4b^2}{4abx-4b^2}=\frac{\frac{1}{3}b^2x^{\frac{-2}{3}}}{4ab}=\frac{\frac{1}{3}b^2x^{\frac{-2}{3}}}{2b^2}=\frac{x^{\frac{-2}{3}}}{6}=\frac{1}{24}$$

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$$\lim_{x\to 8}\frac{8a(\sqrt[3]{x}-2)}{a(x-8)\times 4}\to\lim_{x\to 8}\frac{2(\sqrt[3]{x}-2)}{(\sqrt[3]{x}-2)(\sqrt[3]{x^2}+4+2\sqrt[3]{x})}=\frac{2}{12}=\boxed{\frac{1}{6}}$$