آیلا صفری‌نیا

۱- $f(x) = 2\left[\frac{2-x}{2}\right] + a\left[\frac{x+2}{3}\right]$

$$\left.\begin{array}{l} -2^{+} \to 2(2) - a = 4-a \\ -2^{-} \to 2(1) + 0 = 2 \end{array}\right\} \quad 4-a=2 \to a=2 \qquad \left[\frac{2}{3}\right]=0$$

۲. $\lim_{x\to(\frac{\pi}{6})^-} [2\sin x - 1] \to [2(\frac{1}{2})^- - 1] = [1^- - 1] = [0^-] = -1$

۳. $\lim_{x\to -\frac{1}{2}} [8x^3 - x] = [8(-\frac{1}{2})^3 + \frac{1}{2}] = [8(\frac{-1}{8}) + \frac{1}{2}] = [-1 + \frac{1}{2}] = [\frac{-1}{2}] = -1$

۴. $$\lim_{x\to(-\frac{1}{2})^-} \frac{10x - 5 + [\frac{3}{x^2}]}{16x - [-\frac{2}{x^2}]}$$

$x < -\frac{1}{2} \to x^2 > \frac{1}{4} \to \frac{1}{x^2} < 4 \to \frac{3}{x^2} < 12 \to 11$

$x < \frac{-1}{2} \to x^2 > \frac{1}{4} \to \frac{1}{x^2} < 4 \to -\frac{1}{x^2} > -4 \to \frac{-2}{x^2} > -8 \to -8$

$$\frac{10(-\frac{1}{2})^- + 6}{16(-\frac{1}{2})^- + 8} = \frac{1}{0^-} = -\infty$$

۵. $$\lim_{x\to1^+} \frac{\sin^2\pi x}{[x] + \cos\pi x} = \frac{\sin^2\pi x}{1+\cos\pi x} = \frac{1-\cos^2\pi x}{1+\cos\pi x} = \frac{(1-\cos\pi x)\cancel{(1+\cos\pi x)}}{\cancel{1+\cos\pi x}}$$

$= 1 - \cos\pi x \to 1 - \cos\pi = 1-(-1) = 2$

۶. $$\lim_{x\to-1} \frac{\sqrt{2x+3} - \sqrt{3x+4}}{1+\sqrt[3]{x}} \times \frac{1+\sqrt[3]{x^2} - \sqrt[3]{x}}{1+\sqrt[3]{x^2} - \sqrt[3]{x}} \times \frac{\sqrt{2x+3} + \sqrt{3x+4}}{\sqrt{2x+3} + \sqrt{3x+4}} =$$

$$\frac{-\cancel{(x+1)}(1+\sqrt[3]{x^2} - \sqrt[3]{x})}{\cancel{(1+x)}(\sqrt{2x+3} + \sqrt{3x+4})} = \frac{-(1+\sqrt[3]{x^2} - \sqrt[3]{x})}{\sqrt{2x+3} + \sqrt{3x+4}} = \frac{-(1+1+1)}{2} = \frac{-3}{2}$$

آیلا منزوی‌نیا

۷. $\lim\limits_{x\to 1} \dfrac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}} = \dfrac{2(x-\frac{7}{2}\sqrt{x}+\frac{5}{2})}{2x-\sqrt{3x+1}} = \dfrac{2(\sqrt{x}-1)(\sqrt{x}-\frac{5}{2})}{2x-\sqrt{3x+1}}$

$\times \dfrac{2x+\sqrt{3x+1}}{2x+\sqrt{3x+1}} = \dfrac{2(\sqrt{x}-1)(\sqrt{x}-\frac{5}{2})(2x+\sqrt{3x+1})}{4x^2-3x-1} = \dfrac{(-\frac{3}{2})(4)}{2(2)(\frac{5}{4})} = \dfrac{-\frac{3}{2}}{\frac{5}{4}} = -\dfrac{6}{5}$

$4x^2-3x-1 = 4(x^2-\frac{3}{4}x-\frac{1}{4}) = (x-1)(x+\frac{1}{4})$, $\quad \sqrt{x}+1$

۸. $\lim\limits_{x\to 0} \dfrac{a+\sqrt{bx+c}}{x} = \dfrac{1}{4} \to \lim \dfrac{a+\sqrt{c}}{0} = \dfrac{1}{4}$

$\dfrac{0}{0} \to a+\sqrt{c}=0 \to \sqrt{c}=-a \to c=a^2$, $\quad a<0$

$\to \dfrac{\frac{b}{2\sqrt{bx+c}}}{1} = \dfrac{1}{4} \xrightarrow{x=0} \dfrac{b}{2\sqrt{c}} = \dfrac{1}{4} \to 2b=\sqrt{c} \to 2b=-a$, $\quad -a=\sqrt{c}$, $\quad b=\dfrac{-a}{2}$

$\to \dfrac{ab}{c} = \dfrac{a(\frac{-a}{2})}{a^2} = \dfrac{-a^2}{2a^2} = \dfrac{-1}{2}$

۹. $\lim\limits_{x\to -2\pi} \dfrac{4+k[\frac{x}{\pi}]}{\sin x} = +\infty$

$\xrightarrow{-2\pi^+} \dfrac{4+k[-2^+]}{\sin(-2\pi)^+} = \dfrac{4-2k}{0^+} = +\infty \to 4-2k>0$, $\quad 4>2k \to k<2$

$\xrightarrow{-2\pi^-} \dfrac{4+k[-2^-]}{\sin(-2\pi)^-} = \dfrac{4-3k}{0^-} = +\infty \to 4-3k<0 \to 3k>4 \to k>\dfrac{4}{3}$

$-2<-k<\dfrac{-4}{3} \to [-k]=-2$

۱۰. $\lim\limits_{x\to 8} \dfrac{b\sqrt{2+\sqrt[3]{x}}-2b}{ax-b} \sim 2b-2b=0 \to 2a-b=0 \to b=2a$

$\times \dfrac{b\sqrt{2+\sqrt[3]{x}}+2b}{b\sqrt{2+\sqrt[3]{x}}+2b} = \dfrac{2b^2+b^2x^{\frac{1}{3}}-4b^2}{4abx-4b^2} = \dfrac{\frac{1}{3}b^2x^{-\frac{2}{3}}}{4ab} = \dfrac{\frac{1}{3}b^2x^{-\frac{2}{3}}}{2b^2} = \dfrac{x^{-\frac{2}{3}}}{6} = \dfrac{1}{24}$

$2a=b$