نام: سینا عنایت الحسینی — دوازدهم مشترک A. جمعه

۱-

$$f(x) = 2\left[\frac{2-x}{2}\right] + a\left[\frac{x+2}{3}\right] \qquad \left[\frac{a}{4}\right] = ?!$$

$$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^-} f(x) \longleftarrow \text{در } x=-2 \text{ حد دارد}$$

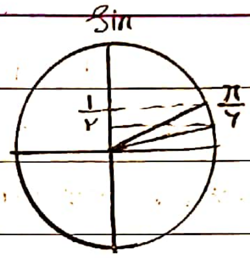
$$2[\overset{1}{\cancel{1^+}}] + a[\overset{a}{\cancel{0^+}}] = 2[\overset{2}{\cancel{2^+}}] + a[\overset{0}{\cancel{0^+}}]$$

$$2 + a = 4 \Rightarrow \boxed{a = 2}$$

۲-

$$\lim_{x\to (\frac{\pi}{6})^-} [2x - 1]$$

$$= \left[2\left(\frac{1}{2}\right)^- - 1\right] = [0^-] = \boxed{-1}$$



۳-

$$\lim_{x\to -\frac{1}{2}} [8x^3 - x]$$

$$\left[8\left(-\frac{1}{8}\right) - \left(-\frac{1}{2}\right)\right] = \left[-1 + \frac{1}{2}\right] = \left[-\frac{1}{2}\right] = \boxed{-1}$$

۴-

$$\lim_{x\to (\frac{-1}{2})^-} \frac{10x - 5 + \left[\frac{3}{x^2}\right]}{14x - \left[-\frac{2}{x^2}\right]}$$

$$\left[\frac{3}{(\frac{1}{4})^+}\right] = [12^-] = 11$$

$$\left[-\frac{2}{(\frac{1}{4})^+}\right] = [-8^-] = -8$$

$$\Rightarrow \lim_{x\to (-\frac{1}{2})^-} \frac{10x - 5 + 11}{14x + 8} = \frac{-5 - 5 + 11}{-8^- + 8} = \frac{1}{0^-} = -\infty$$

$$\lim_{x\to 1^+} \frac{\sin^2 \pi x}{[x]+\cos \pi x} \longrightarrow \lim_{x\to 1^+} \frac{\sin^2 \pi x}{1+\cos \pi x} \longrightarrow$$ -۵

$$\lim_{x\to 1^+} \frac{(\sin \pi x)(\sin \pi x)}{2\cos^2 \frac{\pi x}{2}} \longrightarrow \lim_{x\to 1^+} \frac{\cancel{4}^{\,2} \sin^2 \frac{\pi x}{2} \cancel{\cos^2 \frac{\pi x}{2}}}{\cancel{2} \cancel{\cos^2 \frac{\pi x}{2}}} \longrightarrow$$

$$\lim_{x\to 1^+} 2\sin^2 \frac{\pi x}{2} = \boxed{2}$$

-۶

$$\lim_{x\to -1} \frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}}$$

$$-x-1=-(x+1)$$

$$\lim_{x\to -1} \frac{(2x+3)-(3x+4)}{\cancel{(1+x)}} \cdot \frac{(3)}{(2)} = \boxed{-\frac{3}{2}}$$

-۷

$$\lim_{x\to 1} \frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}} \to \lim_{x\to 1} \frac{2(\sqrt{x}-1)(\sqrt{x}-\frac{5}{2})\,(4)}{4x^2-3x-1}$$

$$\lim_{x\to 1} \frac{\cancel{4}^{\,2}(\cancel{\sqrt{x}-1})(\sqrt{x}-\frac{5}{2})}{\cancel{4}(\cancel{x-1})(x+\frac{1}{4})} \Rightarrow \lim_{x\to 1} \frac{2(\sqrt{x}-\frac{5}{2})}{(\sqrt{x}+1)(x+\frac{1}{4})} = \frac{-3}{\frac{5}{2}} = \frac{-6}{5} = \boxed{-1/2}$$

$$(\cancel{\sqrt{x}-1})(\sqrt{x}+1)$$

-۸ اگر $\lim_{x\to 0} \frac{a+\sqrt{bx+c}}{x} = \frac{1}{4}$ باشد $\frac{ab}{c} = ?$

$x=0$ ریشه صورت است پس: $a+\sqrt{c}=0$ ، $c=a^2$

$$\lim_{x\to 0} \frac{\cancel{a^2}-bx-\cancel{c}}{x(2a)} = \frac{1}{4} \to \lim_{x\to 0} \frac{-b\cancel{x}}{2a\cancel{x}} = \frac{1}{4} \to$$

$$\boxed{b=\frac{-1}{2}a} \Rightarrow \frac{ab}{c} = \frac{a\times(-\frac{1}{2}a)}{a^2} = \boxed{\frac{-1}{2}}$$

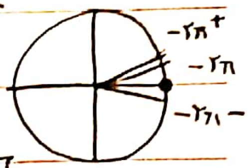
$* \; x > -2\pi \rightarrow \frac{x}{\pi} > -2$

$* \; x < -2\pi \rightarrow \frac{x}{\pi} < -2$

۹-

$$\lim_{x \to -2\pi} \frac{4 + k\left[\frac{x}{\pi}\right]}{\sin x} = +\infty$$

$$-2\pi^+ : \lim_{x \to -2\pi^+} \frac{4 + k\left[\frac{-2\pi^+}{\pi}\right]}{\sin x} \rightarrow \lim_{x \to -2\pi^+} \frac{4 + k\left[\overset{-2}{\cancel{-2^+}}\right]}{\sin x}$$



$$\lim_{x \to -2\pi^+} \frac{(4 - 2k)^{>0}}{(\sin x)_{0^+}} = +\infty \Rightarrow 4 - 2k > 0$$

$$4 > 2k \rightarrow \boxed{k < 2} \quad (I)$$

$$-2\pi^- : \lim_{x \to -2\pi^-} \frac{4 + k\left[\frac{-2\pi^-}{\pi}\right]}{\sin x} \rightarrow \lim_{x \to -2\pi^-} \frac{4 + k[-2^-]}{\sin x}$$

$$\lim_{x \to -2\pi^-} \frac{(4 - 3k)^{<0}}{(\sin x)_{0^-}} = +\infty \qquad 4 - 3k < 0$$

$$4 < 3k \rightarrow \frac{4}{3} < k$$

$$(I) \cap (II) : \frac{4}{3} < k < 2 \Rightarrow -2 < -k < \frac{-4}{3} \Rightarrow \boxed{[-k] = -2}$$

۱۰-

$$\lim_{x \to 8} \frac{b\sqrt{2 + \sqrt[3]{x}} - 2b}{ax - b} \quad \frac{0}{0} \qquad \boxed{8a = b} \; *$$

$$b^2(-2 + \sqrt[3]{x})$$

$$\lim_{x \to 8} \frac{\overbrace{b^2(2 + \sqrt[3]{x}) - 4b^2}^{2b^2 - b^2\sqrt[3]{x}}}{(ax - b)(4a)} \rightarrow \lim_{x \to 8} \frac{-2b^2 + \sqrt[3]{x}\, b^2}{(ax - b)(4\cancel{b})} \; *$$

$$\lim_{x \to 8} \frac{b(\sqrt[3]{x} - 2)}{ax - 8a} \rightarrow \lim_{x \to 8} \frac{\overset{8a}{b}\,\cancel{(\sqrt[3]{x} - 2)}}{4a(x - 8)}$$

$$(\cancel{\sqrt[3]{x} - 2})(\sqrt[3]{x^2} + 2\sqrt[3]{x} + 4)$$

$$\Rightarrow \lim_{x \to 8} \frac{2}{\sqrt[3]{x^2} + 2\sqrt[3]{x} + 4} = \frac{2}{12} = \boxed{\frac{1}{6}}$$