نگین مکری

دوازدهم ریاضی A　　　　تکلیف ۱۹

۱- وقتی عبارتی هم داریم براکت جزء صحیح، حد چپ و راست می‌گیریم

$\lim\limits_{x\to -2} f(x)$

-2^+: $x>-2$

$x>-2$
$-x<2$
$2-x<4$
$\frac{2-x}{2}<2 \to \left[\frac{2-x}{2}\right]=1$

$x+2>0$
$\frac{x+2}{3}>0 \to \left[\frac{x+2}{3}\right]=0$

$f(x)=2+a(0)=2$

-2^-: $x<-2$

$x<-2$
$-x>2$
$2-x>4$
$\frac{2-x}{2}>2$
$\left[\frac{2-x}{2}\right]=2$

$x+2<0$
$\frac{x+2}{3}<0$
$\left[\frac{x+2}{3}\right]=-1$

$f(x)=4-a$

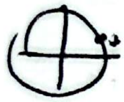
$4-a=2$
$2=a \to \left[\frac{a}{3}\right]=\left[\frac{2}{3}\right]=0$

۲-

$$\lim_{x\to(\frac{\pi}{6})^-}[2\sin x-1]=[2(\tfrac{1}{2})-1]=[0^-]=-1$$

می‌دانیم $\sin x$ در نواحی کمتر از $\frac{\pi}{6}$ کمتر از $\frac{1}{2}$ است

$\sin\frac{\pi}{6}=\frac{1}{2}$



۳-

$$\lim_{x\to-\frac{1}{2}}[8x^3-x]=[8\times(-\tfrac{1}{8})-(-\tfrac{1}{2})]=[-\tfrac{1}{2}]=-1$$

۴-

$$\lim_{x\to(-\frac{1}{2})^-}\frac{10x-5+[\frac{3}{x^2}]}{16x-[-\frac{2}{x^2}]}=\frac{10(-\frac{1}{2})-5+11}{16x+8}$$

$x<-\frac{1}{2}$
$x^2>\frac{1}{4}$
$\frac{1}{x^2}<4 \to \frac{3}{x^2}<12$
$\left[\frac{3}{x^2}\right]=11$

$\frac{2}{x^2}<8$
$\frac{-2}{x^2}>-8$
$\left[\frac{-2}{x^2}\right]=-8$

$$=\frac{-5-5+11}{16(-\frac{1}{2})+8}=\frac{1}{0^-}=-\infty$$

۵-

$$\lim_{x\to1^+}\frac{\sin^2\pi x}{[x]+\cos\pi x}=\frac{\sin^2\pi}{1+\cos\pi}=\frac{4\sin^2\frac{\pi}{2}\cancel{\cos^2\frac{\pi}{2}}}{2\cancel{\cos^2\frac{\pi}{2}}}=2\sin^2\frac{\pi}{2}=2$$

$[x]=1$

$\sin\pi=2\sin\frac{\pi}{2}\cos\frac{\pi}{2}$

$\sin^2\pi=4\sin^2\frac{\pi}{2}\cos^2\frac{\pi}{2}$

۶-

$$\lim_{x\to -1} \frac{\sqrt{2x+3} - \sqrt[3]{3x+4}}{1+\sqrt[3]{x}} \quad \frac{0}{0} \text{ (ابهام)} \quad \overset{HOP}{=} \quad \frac{\frac{2}{2\sqrt{2x+3}} - \frac{3}{3\sqrt[3]{(3x+4)^2}}}{\frac{1}{3\sqrt[3]{x^2}}} = \frac{-\frac{1}{2}}{\frac{1}{3}} = -\frac{3}{2}$$

۷-

$$\lim_{x\to 1} \frac{2x - 7\sqrt{x} + 5}{2x - \sqrt{3x+1}} = \frac{0}{0} \text{ (ابهام)} \quad \overset{HOP}{=} \quad \frac{2 - 7\left(\frac{1}{2\sqrt{x}}\right)}{2 - \left(\frac{3}{2\sqrt{3x+1}}\right)} = \frac{2 - \frac{7}{2}}{\frac{5}{4}} = \frac{-\frac{3}{2}}{\frac{5}{4}}$$

$$= \frac{-12}{10} = -\frac{6}{5}$$

۸-

$$\lim_{x\to 0} \frac{a + \sqrt{bx+c}}{x} = \frac{1}{4} \Rightarrow \frac{b}{2\sqrt{bx+c}} = \frac{1}{4} \Rightarrow \frac{b}{2\sqrt{c}} = \frac{1}{4}$$

حتماً از روش رفع ابهام است.

$4b = 2\sqrt{bx+c}$

$2b = \sqrt{bx+c}$

$4b = 2\sqrt{c}$

$2b = \sqrt{c}$

$\sqrt{bx+c} = -a$

$2b = -a$

$2b = \sqrt{c}$

$4b^2 = c$

$$\Rightarrow \frac{ab}{c} = \frac{-2b \times b}{4b^2} = -\frac{1}{2}$$

۹-

$$\frac{4 + k\left[\frac{x}{\pi}\right]}{\sin x} = +\infty$$

$x \to -2\pi^+$: $\sin x = 0^+$

$4 + k\left[\frac{x}{\pi}\right] > 0 \rightarrow 4 - 2k > 0$

$4 > 2k$

$2 > k$ (1)

$x \to -2\pi^-$: $\sin x = 0^-$

$4 + k\left[\frac{x}{\pi}\right] < 0$ $\quad 4 - 3k < 0$

$4 < 3k$

$\frac{4}{3} < k$ (1)

$$\Rightarrow \frac{4}{3} < k < 2$$

$$-2 > -k > -\frac{4}{3} \rightarrow [-k] = -2$$

۱۰-

$$\lim_{x\to 8} \frac{b\sqrt{2+\sqrt[3]{x}} - 2b}{ax - b} = \frac{0}{0}$$

$$\frac{2b - 2b}{8a - b} = \frac{0}{0}$$

$8a - b = 0$

$8a = b \rightarrow \frac{b}{a} = \frac{8a}{a} = 8$

$$\overset{HOP}{\rightsquigarrow} \quad \frac{b \times \frac{\frac{1}{3\sqrt[3]{x^2}}}{2\sqrt{2+\sqrt[3]{x}}}}{a} = \frac{b}{48a} = \frac{b}{a} \times \frac{1}{48} = \frac{1}{6}$$