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دوازدهم دفتر A

پرنیان شمشیری ـ تکلیف شماره ۱۹

سوال ۱)

$x=-2$:

$-2^+ : \lim_{x\to -2^+} 2\left[\frac{2-x}{2}\right]+a\left[\frac{x+2}{3}\right] = 2[2^-]+a[0^+] = 2(1)+0=2$

$-2^- : \lim_{x\to -2^-} 2\left[\frac{2-x}{2}\right]+a\left[\frac{x+2}{3}\right] = 2[2^+]+a[0^-] = 2(2)+(-a) = 4-a$

$\left\{ 4-a=2 \right. \to a=2$

$\to \left[\frac{a}{3}\right] = \left[\frac{2}{3}\right] = 0$

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سوال ۲)

$\lim_{x\to (\frac{\pi}{6})^-} [2\sin x-1] \to x<\frac{\pi}{6} \to \sin x<\frac{1}{2} \to 2\sin x<1 \to 2\sin x-1<0$

$\to \lim_{x\to (\frac{\pi}{6})^-}[2\sin x-1] = \lim[0^-] = -1$

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سوال ۳)

$\lim_{x\to -\frac{1}{2}} [8x^3-x] = \lim\left[8\left(-\frac{1}{8}\right)-\left(-\frac{1}{2}\right)\right] = \lim\left[-1+\frac{1}{2}\right] = \lim\left[-\frac{1}{2}\right] = -1$

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سوال ۴)

$\lim_{x\to (-\frac{1}{2})^-} \frac{10x-5+\left[\frac{3}{x^2}\right]}{14x-\left[\frac{-2}{x^2}\right]} = \frac{-5-5+12}{-8-7} = \frac{2}{-15}$

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$x<-\frac{1}{2} \to x^2>\frac{1}{4} \to \frac{3}{x^2}>\frac{3}{\frac{1}{4}} \to \frac{3}{x^2}>12 \to \left[\frac{3}{x^2}\right]=12$

$x<-\frac{1}{2} \to x^2>\frac{1}{4} \to \frac{2}{x^2}>\frac{2}{\frac{1}{4}} \to \frac{2}{x^2}>8 \to \frac{-2}{x^2}<8 \to \left[\frac{-2}{x^2}\right]=7$

سوال ۵)

$\lim_{x\to 1^+} \frac{\sin^2 \pi x}{[x]+\cos \pi x} = \frac{0}{1+(-1)} = \frac{0}{0}$ رفع ابهام $= \frac{1-\cos^2 \pi x}{1+\cos \pi x} = \frac{(1+\cos \pi x)(1-\cos \pi x)}{1+\cos \pi x} = 1-(-1) = 2$

$x>1 \to \pi x>1 \to \sin^2 \pi x<1 \to \sin^2 \pi x \simeq 0$

$x>1 \to \pi x>1 \to \cos \pi x \neq 1 \to \cos \pi \simeq -1$

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سوال ۶)

$$\lim_{x\to -1}\frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\times\frac{\sqrt{2x+3}+\sqrt{3x+4}}{\sqrt{2x+3}+\sqrt{3x+4}}\times\frac{1+\sqrt[3]{x^2}-\sqrt[3]{x}}{1+\sqrt[3]{x^2}-\sqrt[3]{x}}=\frac{\overbrace{\left((2x+3)-(3x+4)\right)}^{2x+3-3x-4=-x-1=-(x+1)}\left(1+\sqrt[3]{x^2}-\sqrt[3]{x}\right)}{\left(\sqrt{2x+3}+\sqrt{3x+4}\right)(1+x)}$$

$$=\frac{(-1)(1+1-(-1))}{(1+1)}=\frac{(3)(-1)}{2}=-\frac{3}{2}$$

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سوال ۷)

$$\lim_{x\to 1}\frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\times\frac{2x+\sqrt{3x+1}}{2x+\sqrt{3x+1}}\times\frac{2x+5+7\sqrt{x}}{2x+5+7\sqrt{x}}=\frac{(4x^2+25+20x-49x)(2x+\sqrt{3x+1})}{(4x^2-3x-1)(2x+5+7\sqrt{x})}$$

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$$=\frac{(4x^2-29x+25)(2x+\sqrt{3x+1})}{(4x^2-3x-1)(2x+5+7\sqrt{x})}=\frac{\cancel{(x-1)}(4x-25)(2x+\sqrt{3x+1})}{\cancel{(x-1)}(4x+1)(2x+5+7\sqrt{x})}=\frac{(-21)(2+2)}{(-3)(14)}=\frac{-21\times 4}{-3\times 14}=2$$

سوال ۸)

$$\lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x}=\frac{0}{0}\to a+\sqrt{c}=0\to a=-\sqrt{c}\to a^2=c$$

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$$\xrightarrow{\text{رفع ابهام}}\frac{a^2-bx-c}{(x)(a-\sqrt{bx+c})}\xrightarrow{a^2=c}\frac{-b\cancel{x}}{\cancel{(x)}(a-\sqrt{bx+c})}=\frac{-b}{a-\sqrt{bx+c}}\xrightarrow{x=0}\frac{-b}{a-\sqrt{c}}=\frac{1}{4}\xrightarrow{a=-\sqrt{c}}\frac{-b}{2a}=\frac{1}{4}$$

$$\longrightarrow b=\frac{-1}{2}a\ ,\ c=a^2\to\frac{ab}{c}=\frac{a\times\frac{-1}{2}a}{a^2}=\frac{-1}{2}$$

سوال ۴)

$$x\to\left(-\frac{1}{2}\right)^-\longrightarrow x<-\frac{1}{2}\to x^2>\frac{1}{4}\to\frac{1}{x^2}<4\to\frac{3}{x^2}<12\to\left[\frac{3}{x^2}\right]=11$$

$$\frac{1}{x^2}<4\to-\frac{1}{x^2}>-4\to\frac{-2}{x^2}>-8\to\left[\frac{-2}{x^2}\right]=-8$$

$$\to\lim_{x\to(-\frac{1}{2})^-}=\frac{10x-5+11}{16x-(-8)}=\lim_{x\to(-\frac{1}{2})^-}=\frac{10x+6}{16x+8}=\frac{-5+6}{(-8)^-+8}=\frac{1}{0^-}=-\infty$$

سوال ۷)

$$\lim_{x\to 1}\frac{(\sqrt{x}-1)(2\sqrt{x}-5)}{2x-\sqrt{3x+1}}=\frac{(\sqrt{x}-1)(2\sqrt{x}-5)}{2x-\sqrt{3x+1}}\times\frac{2x+\sqrt{3x+1}}{2x+\sqrt{3x+1}}\longrightarrow$$

$$\lim_{x\to 1}\frac{(\sqrt{x}-1)(2\sqrt{x}-5)\times(2x+\sqrt{3x+1})}{4x^2-3x-1}=\lim_{x\to 1}\frac{(\sqrt{x}-1)(2\sqrt{x}-5)(4)}{(4x+1)(x-1)}=\ (\sqrt{x}-1)(\sqrt{x}+1)$$

$$\lim_{x\to 1}\frac{(2\sqrt{x}-5)(4)}{(4x+1)(\sqrt{x}+1)}=\frac{(-3)(4)}{5\times 2}=-\frac{1}{2}$$

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سوال ۹

$$\lim_{x\to(-۲\pi)^-} = \frac{۴+K\left[\frac{x}{\pi}\right]}{\sin x} = \frac{۴-۳K}{۰^-} = +\infty \to ۴-۳K<۰ \to K>\frac{۴}{۳} \quad (I)$$

$$\lim_{x\to(-۲\pi)^+} = \frac{۴+K\left[\frac{x}{\pi}\right]}{\sin x} = \frac{۴-۲K}{۰^+} = +\infty \to ۴-۲K>۰ \to K<۲ \quad (II)$$

$$\xrightarrow{(I)\cap(II)} \frac{۴}{۳}<K<۲ \to -۲<-K<-\frac{۴}{۳} \to [-K] = -۲$$

سوال ۱۰

$$۸a-b=۰ \to b=۸a$$

$$\lim_{x\to ۸} \frac{۸a\left(\sqrt{۲+\sqrt[۳]{x}}-۲\right)}{ax-۸a} = \frac{۸a\left(\sqrt{۲+\sqrt[۳]{x}}-۲\right)}{ax-۸a} \times \frac{\sqrt{۲+\sqrt[۳]{x}}+۲}{\sqrt{۲+\sqrt[۳]{x}}+۲} =$$

$$\lim_{x\to ۸} \frac{۸a\left(\sqrt[۳]{x}-۲\right)}{a(x-۸)\times ۴} \to \lim_{x\to ۸} \frac{۲\left(\sqrt[۳]{x}-۲\right)}{\left(\sqrt[۳]{x}-۲\right)\left(\sqrt[۳]{x^۲}+۴+۲\sqrt[۳]{x}\right)} = \frac{۲}{۱۲} = \frac{۱}{۶}$$