کیمیاییان آبالی

۱

$f(x)=2\left[\frac{2-x}{2}\right]+a\left[\frac{x+2}{3}\right] \xrightarrow[x\to -2^+]{} \frac{2-x}{2}<2$ و $\frac{x+2}{3}>0$ $\quad \lim_{x\to -2^+} f(x)= 2\underbrace{[2^-]}_{1}+a\times\underbrace{[0^+]}_{0}=2$

$\xrightarrow{x<-2} \quad \frac{2-x}{2}>2$ و $\frac{x+2}{3}<0 \quad \lim_{x\to -2^-} f(x) = 2\underbrace{[2^+]}_{2}+a\underbrace{[0^-]}_{-1}=4-a$

$\left[\frac{\overset{2}{\cancel{a}}}{3}\right]=0$

$\boxed{a=2} \leftarrow 4-a=2 \leftarrow$ چون در $x=-2$ حد دارد بنابراین حد چپ و راست برابر باشد

۲

$\lim_{x\to(\frac{\pi}{4})^-}[2\sin x-1]$

$x<\frac{\pi}{4} \to \sin x<\frac{1}{2} \to 2\sin x<1 \to 2\sin x-1<0$ ★

طبق ★ $= \lim_{x\to(\frac{\pi}{4})^-}[0^-] = -1$

۳

$\lim_{x\to -\frac{1}{2}}[8x^3-x] = \lim_{x\to -\frac{1}{2}}\left[\underbrace{\underbrace{8\cancel{\left(-\frac{1}{2}\right)^3}}_{-1}\underbrace{\cancel{-\left(-\frac{1}{2}\right)}}_{+}}_{-\frac{1}{2}}\right] = -1$

$-\frac{1}{2} \to$ جواب عدد صحیح نبود $\to$ ~~نیاز به بررسی چپ و راست~~

۴

طبق ★

$\lim_{x\to(-\frac{1}{2})^-}\frac{10x-5+\left[\frac{3}{x^2}\right]}{14x-\left[-\frac{2}{x^2}\right]} = \lim_{x\to(-\frac{1}{2})^-}\frac{1}{0} = -\infty$

$x<-\frac{1}{2} \to x^2>\frac{1}{4} \to \frac{3}{x^2}<12 \xrightarrow{\text{صورت کسر}} \underbrace{10\left(-\frac{1}{2}\right)}_{-5}-5+11=1$ ★

$-\frac{2}{x^2}>-8$; مخرج کسر: $14x-\underbrace{\left[-\frac{2}{x^2}\right]}_{-8} = 14x+8<0$ ★

$14x<-8$

۵

$\lim_{x\to 1^+}\frac{\sin^2\pi x}{[x]+\cos\pi x} = \lim_{x\to 1^+}\frac{\cancel{(\sin^2\pi x)}(1-\cos\pi x)}{\underbrace{(1+\cos\pi x)(1-\cos\pi x)}_{\cancel{1-\cos^2\pi x}}} = \lim_{x\to 1^+} 1-\cos\overset{-1}{\pi x} = 2$

۶

$\lim_{x\to -1}\frac{\left(\sqrt{2x+3}-\sqrt{3x+4}\right)\left(\sqrt{2x+3}+\sqrt{3x+4}\right)\left(1+\sqrt[3]{x^2}-\sqrt[3]{x}\right)}{\left(1+\sqrt[3]{x}\right)\left(\sqrt{2x+3}+\sqrt{3x+4}\right)\left(1+\sqrt[3]{x^2}-\sqrt[3]{x}\right)} = \frac{\overset{-1}{\cancel{(2x+3-3x-4)}}\left(1+\sqrt[3]{x^2}-\sqrt[3]{x}\right)}{\cancel{(1+x)}\left(\sqrt{2x+3}+\sqrt{3x+4}\right)}$

$= -\frac{(1+1+1)}{(1+1)} = -\frac{3}{2}$

7

$$\lim_{x\to 1}\frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}} \xrightarrow{\text{مشتق}} \frac{2-\frac{7}{2}}{2-\frac{3}{2\sqrt{4}}} = \frac{-\frac{3}{2}}{\frac{5}{4}} = -\frac{6}{5}$$

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8 تابع صفر صفر بوده که ساده نشده است

$$\lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x}=\frac{1}{4} \rightarrow a+\sqrt{c}=0 \rightarrow -a=\sqrt{c}$$

$$\frac{b}{2\sqrt{c}}=\frac{1}{4} \rightarrow 2b=\sqrt{c} \qquad \frac{a\times b}{c} = -\frac{c}{2c} = -\frac{1}{2}$$

9

$$\lim_{x\to -2\pi}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}=+\infty$$

$$x>-2 \rightarrow x>-2\pi \rightarrow \frac{x}{\pi}>-2$$

$$\frac{4+k\,\cancel{[-2^+]}\,-2}{0^+}=+\infty,\quad >0 \longrightarrow 2k<4,\ k<2$$

$$x<-2 \rightarrow x<-2\pi \rightarrow \frac{x}{\pi}<-2$$

$$\frac{4+k\,\cancel{[-2^-]}\,-3}{0^-}=+\infty,\quad <0 \longrightarrow 3k>4,\ k>\frac{4}{3}$$

$$\frac{4}{3}<k<2 \rightarrow -2<-k<-\frac{4}{3}$$

$$[-k]=-2$$

10

$$\lim_{x\to 8}\frac{b\sqrt{2+\sqrt[3]{x}}-2b}{ax-b}$$

$8a+b=0$

$b=-8a$

$$\xrightarrow{\text{مشتق}} \frac{-8\cancel{a}\times\frac{\frac{1}{3\times 4}}{2\sqrt{4}}}{\cancel{a}} = \frac{-8}{48} = -\frac{1}{6}$$