

1, 2

جواب

$$\lim_{x \rightarrow 0} \frac{x^2 - 1}{x - 1} = \frac{-1}{-1} = 1 \Rightarrow x^2 + ax + b = (x - 1)^2 = x^2 - 2x + 1 \Rightarrow a + b = -2 + 1 = -1$$

2

$$\lim_{x \rightarrow \infty} \frac{x^2 - 1}{x - 1} = \frac{\infty}{\infty} \Rightarrow x^2 + ax + b = (x - \sqrt{x})^2 = x^2 - 2\sqrt{x} + x \Rightarrow \left[\frac{b}{a} \right] = \left[\frac{\sqrt{x}}{-\sqrt{x}} \right] = \left[\frac{1}{-1} \right] = -1$$

2

$$\lim_{x \rightarrow \infty} \frac{\left[\frac{1}{\sqrt{x}} \right] + a}{\left[\frac{1}{\sqrt{x}} \right] + a} = -\infty \Rightarrow \lim_{x \rightarrow \infty} \frac{a + \frac{1}{\sqrt{x}}}{a + \frac{1}{\sqrt{x}}} = -\infty \Rightarrow a = -\frac{1}{\sqrt{x}} \Rightarrow a = -\frac{1}{\sqrt{x}} \Rightarrow [a] = -1$$

$$\Rightarrow \frac{a - 1}{a + \frac{1}{\sqrt{x}}} = -\infty$$

$$a < 0 \Rightarrow \frac{a - 1}{a + \frac{1}{\sqrt{x}}} = -\infty$$

2

$$x > -\frac{1}{r} \rightarrow x' < \frac{1}{r} \rightarrow \frac{1}{x'} > r \rightarrow \left[\frac{r}{x'} \right] = 1r$$

سوال 1

$$\frac{1}{x'} > r \rightarrow \frac{-r}{x'} < -1 \rightarrow \left[\frac{-r}{x'} \right] = -1$$

$$\lim_{x \rightarrow (-\frac{1}{r})^+} \frac{15x + 9}{x^2 + 12} = \lim_{x \rightarrow (-\frac{1}{r})^+} \frac{-1 + 9}{0^+} = \frac{1}{0^+} = +\infty$$

سوال ۲

$$\lim_{x \rightarrow -1} \frac{2x^2 + 10x + 12}{1x + 4\sqrt{x}} = \lim_{x \rightarrow -1} \frac{(2x+1)(x+2)}{x(x+\sqrt{x})} \times \frac{\sqrt{x^2+x} - x\sqrt{x}}{\sqrt{x^2+x} - x\sqrt{x}} \rightarrow$$

$$\lim_{x \rightarrow -1} \frac{(2x+1)(x+2) \times 1x}{x(x+\sqrt{x})} \rightarrow \lim_{x \rightarrow -1} \frac{1x(2x+2)}{4} = 2 \times (-4) = -12$$

$$\Rightarrow \frac{14n + 1 \left[\frac{1}{n^2} \right]}{18 + 1 \left[\frac{1}{n^2} \right]} = \frac{1 \left(14n + \left[\frac{1}{n^2} \right] \right)}{1 \left(18 + \left[\frac{1}{n^2} \right] \right)} = \frac{1}{18}$$

-8

$$\lim_{n \rightarrow \infty} \frac{x^n - 8}{n^n - [8^n]} = \frac{(x-8)(n+1)}{(n-1)(n^2 + 8n)} = \frac{x-8}{8+8+8} = \frac{x-8}{24} = \frac{1}{12} \checkmark$$

-5

$$\Rightarrow \frac{14n+10}{9n^2} = \frac{14n+1}{9n^2} \Rightarrow \lim_{n \rightarrow -1} \frac{14n+1}{9n^2} = \frac{-9}{9} = -1$$

-4

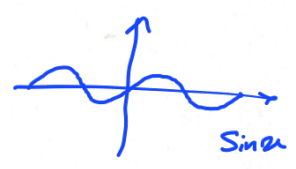
$$\lim_{x \rightarrow 0^-} \frac{\sqrt{1+\cos x} - \sqrt{1-\cos x}}{\sqrt{1-\cos x}} \xrightarrow{\text{L'Hopital}} \lim_{x \rightarrow 0^-} \frac{\frac{1}{2\sqrt{1+\cos x}} - \frac{1}{2\sqrt{1-\cos x}}}{\frac{1}{2\sqrt{1-\cos x}}} = \frac{\frac{1}{2\sqrt{1+1}} - \frac{1}{2\sqrt{1-1}}}{\frac{1}{2\sqrt{1-1}}} = \frac{\frac{1}{2\sqrt{2}} - \frac{1}{0}}{\frac{1}{0}} = -\frac{1}{\sqrt{2}}$$

-1

$$\lim_{x \rightarrow 0^-} \frac{\sqrt{1+\cos x} - \sqrt{1-\cos x}}{\sqrt{1-\cos x}} \times \frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} + \sqrt{1-\cos x}} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} \rightarrow \lim_{x \rightarrow 0^-} \frac{1+\cos x - 1 + \cos x}{\sqrt{1-\cos x}(\sqrt{1+\cos x} + \sqrt{1-\cos x})} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} =$$

سوال ۳

$$\lim_{x \rightarrow 0^-} \frac{\cos x \times \sqrt{x}}{|\sin x| \times \sqrt{x}} = \lim_{x \rightarrow 0^-} \frac{\cos x \times \sqrt{x}}{-\sqrt{x} \sin x} \xrightarrow{\text{L'Hopital}} \lim_{x \rightarrow 0^-} \frac{-\sin x}{-\cos x} \times \frac{\frac{1}{2\sqrt{x}}}{-\frac{1}{2\sqrt{x}}} = -1$$



$$\lim_{n \rightarrow \infty} \frac{k + 1 - \frac{a n^p}{p}}{k n^p} = p \Rightarrow \frac{p k + p - a n^p}{p k n^p} = p \xrightarrow{\text{hoplim}} \frac{-p a n}{p k n} \Rightarrow \lim_{n \rightarrow \infty} \frac{-a}{k} = p - 1$$

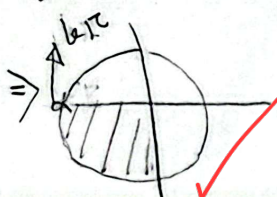
$$\Rightarrow \frac{a}{k} = p - 1 = -9 \quad \checkmark \quad (Y)$$

$$\lim_{n \rightarrow \infty} \frac{p \sqrt{n-1/a} + p \sqrt{n} - p \sqrt{1/a}}{\sqrt{(n-1/a)(n+1/a)}} \Rightarrow \frac{(\sqrt{n} - \sqrt{1/a})^p}{p \sqrt{n} - p \sqrt{1/a}} = \frac{\sqrt{n^p} + \sqrt{p} \sqrt{a} \sqrt{n} \sqrt{1/a} + 9 a \sqrt{n}}{(n-1/a)^{p/2} + 9 a \sqrt{n}}$$

$$\Rightarrow \frac{(\sqrt{n-1/a})^p}{\sqrt{(n-1/a)(n+1/a)}} \Rightarrow \frac{(\sqrt{n-1/a})^p}{\sqrt{(n+1/a)(n-1/a)}} \Rightarrow \frac{p}{\sqrt{1/a}}$$

$$\lim_{n \rightarrow \infty} \left(\frac{p}{\sqrt{1/a}} + \frac{p \sqrt{n-1/a}}{(\sqrt{n+1/a})(\sqrt{n-1/a})} \right) = \lim_{n \rightarrow \infty} \frac{p}{\sqrt{1/a}} + 0 = \frac{\sqrt{p}}{\sqrt{1/a}} = \sqrt{\frac{p}{1/a}} = \sqrt{\frac{p a}{1}} = \sqrt{p a}$$

$$\lim_{n \rightarrow -1} \frac{1 - k[n]}{n^p - 1} = -\infty$$



$$n \rightarrow -1^+ \quad 1 + \frac{k}{n} = -\infty \Rightarrow k + 1 > 0 \Rightarrow k > -1$$

$$n \rightarrow -1^- \quad 1 + \frac{k}{n} = -\infty \Rightarrow p k + 1 < 0 \Rightarrow p k < -1 \Rightarrow k < -\frac{1}{p}$$

$$\Rightarrow -1 < k < -\frac{1}{p}$$

