

$$\lim_{x \rightarrow 2} \frac{2x-2}{x^2+ax+b} = -\infty$$

$$2x-2 \xrightarrow{x \rightarrow 2} 2(2)-2 = -1 \rightarrow \text{خرج} = 0^+ \text{ باز} \text{ } 2, 2^+$$

$$(x-2)^2 = x^2+ax+b$$

$$\begin{cases} a = -4 \\ b = 4 \end{cases} \rightarrow a+b = 0$$

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$$\lim_{x \rightarrow \sqrt[3]{4}} \frac{x}{x^2+ax+b} = +\infty$$

$$x \xrightarrow{x \rightarrow \sqrt[3]{4}} x = \sqrt[3]{4} \Rightarrow \text{خرج} = 0^+ \quad (\sqrt[3]{4})^-, (\sqrt[3]{4})^+ \text{ باز}$$

$$(x-\sqrt[3]{4})^2 = x^2+ax+b$$

$$\begin{cases} a = -2\sqrt[3]{4} = \sqrt[3]{-32} \\ b = \sqrt[3]{16} \end{cases} \rightarrow \left[\frac{b}{a} \right] = \left[\sqrt[3]{\frac{16}{-32}} \right] = \boxed{-1}$$

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$$\lim_{x \rightarrow (\frac{1}{\sqrt{e}})^+} \frac{[-x]+a}{\sqrt[3]{\frac{1}{e}}x+a} = -\infty$$

$$\begin{aligned} a > -\frac{1}{\sqrt{e}} &\Rightarrow \frac{a-1}{\sqrt[3]{a}+\frac{1}{\sqrt{e}}} = -\infty \Rightarrow a = \left(-\frac{1}{\sqrt{e}}\right)^3 \Rightarrow \frac{-\sqrt[3]{e}}{0^+} = -\infty \\ a < -\frac{1}{\sqrt{e}} &\Rightarrow \frac{a-1}{\sqrt[3]{a}-\frac{1}{\sqrt{e}}} = -\infty \end{aligned}$$

$$x = \frac{1}{\sqrt{e}} \Rightarrow -x = -\frac{1}{\sqrt{e}} \Rightarrow [-x] = -1 \Rightarrow \frac{a-1}{a+\frac{1}{\sqrt{e}}+a} \Rightarrow [a] = \left[-\frac{1}{\sqrt{e}}\right] = \boxed{-1}$$

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$$\lim_{x \rightarrow (-\frac{1}{\sqrt{e}})^+} \frac{14x - \left[-\frac{2}{x^2}\right]}{x^2x + \left[\frac{5}{x^2}\right]}$$

$$\begin{aligned} x > -\frac{1}{\sqrt{e}} &\Rightarrow x^2 < \frac{1}{e} \Rightarrow \frac{1}{x^2} > e \Rightarrow \begin{cases} -\frac{2}{x^2} < -1 \Rightarrow \left[-\frac{2}{x^2}\right] = -9 \\ \frac{5}{x^2} > 12 \Rightarrow \left[\frac{5}{x^2}\right] = 12 \end{cases} \rightarrow \frac{14x - (-9)}{x^2x + 12} = \frac{-14+9}{-12^++12} = \frac{-5}{0^+} = \boxed{+\infty} \end{aligned}$$

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$$\lim_{x \rightarrow 2^+} \frac{x^2-4}{x^2-[2^x]} \xrightarrow{\frac{0}{0}} \frac{(x-2)(x+2)}{(x-2)(x^2+2x+4)} = \frac{4}{12} = \boxed{\frac{1}{3}}$$

$$x^2 = 4^+ \Rightarrow [x^2] = 4$$

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$$\lim_{x \rightarrow -1} \frac{x^2 + \ln x + 1}{1 + 9\sqrt{x}} = \frac{1 - 1 + 1}{1 - 1} = \frac{0}{0}$$

$$\Rightarrow \frac{x^2 + \ln x + 1}{9(1 + \sqrt{x})} \times \frac{\sqrt{x^2 - 1} + 1}{\sqrt{x^2 - 1} + 1} = \frac{(x+1)(x-1)(\sqrt{x^2 - 1} + 1)}{9(A+x)} \xrightarrow{x=-1} \frac{-9 \times 1}{9} = (-1)$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1-\cos x}} = \frac{\sqrt{1}-\sqrt{1}}{\sqrt{1-1}} = \frac{0}{0} \rightarrow \text{L'Hôpital's Rule}$$

$$\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1-\cos x}} \times \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \times \frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} + \sqrt{1-\cos x}} = \frac{(1+x-x)(\sqrt{1+x} + \sqrt{1-x})(\sqrt{1+\cos x} + \sqrt{1-\cos x})}{(-\sin x)(\sqrt{1+x} + \sqrt{1-x})(\sqrt{1+\cos x} + \sqrt{1-\cos x})} \xrightarrow{x=0} \frac{-1 \times \sqrt{2}}{1 \times \sqrt{2}} = (-1)$$

$\sqrt{1-\cos x} = \sqrt{2\sin^2 \frac{x}{2}} = |\sin \frac{x}{2}| = |\sin 0| = -\sin \frac{x}{2}$

$$\lim_{x \rightarrow 0} \frac{K + \cos \sqrt{a}x}{Kx^2} = \frac{K+1 - \frac{ax^2}{2}}{Kx^2} \xrightarrow{\text{L'Hôpital}} \frac{K+1 - \frac{ax^2}{2}}{Kx^2} = \frac{K+1}{K} = \frac{1}{K}$$

$\cos x \approx 1 - \frac{x^2}{2}$

$$\frac{K+1}{K} = \frac{1}{K} \Rightarrow K+1 = 1 \Rightarrow K = -1$$

$$\frac{a}{K} = \frac{1}{-1} = -1$$

$$\lim_{x \rightarrow \sqrt{a}} \frac{\sqrt{x-\sqrt{a}} + \sqrt{x} - \sqrt{a}}{\sqrt{x^2 - 9a}} = \frac{0}{0} \rightarrow \text{L'Hôpital's Rule}$$

$$\frac{\frac{1}{2\sqrt{x-\sqrt{a}}} + \frac{1}{2\sqrt{x}}}{\frac{2x}{\sqrt{x^2 - 9a}}} = \frac{1}{2\sqrt{x-\sqrt{a}}} + \frac{1}{2\sqrt{x}} \times \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}} = \frac{1}{2\sqrt{x-\sqrt{a}}} + \frac{1}{2\sqrt{x}} \times \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}}$$

$$x = \sqrt{a} \Rightarrow \frac{1}{2\sqrt{0}} + \frac{1}{2\sqrt{a}} = \frac{1}{2\sqrt{a}}$$

$$\lim_{x \rightarrow -1} \frac{1 - K[x]}{x^2 - 1} = \frac{1+K}{0^-} = -\infty \Rightarrow K > -1$$

$$\frac{1+K}{0^+} = -\infty \Rightarrow K < -1$$

